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INTEGRATING MACHINE LEARNING WITH BAROCLINIC INSTABILITY MODELS FOR ADVANCED MESOSCALE ENERGY CASCADE ANALYSIS

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Abstract

This study combines traditional methods, like normal mode decomposition, with cutting-edge machine learning (ML) techniques to enhance the analysis of energy transfer across scales in baroclinic instability models. By leveraging high-resolution, non-hydrostatic simulations, we explore the energy distribution between geostrophic and ageostrophic modes, uncovering distinctive spectral slopes of -3.1 and -2.7, respectively, which underscore the role of inertia-gravity waves at the mesoscale. Employing Convolutional Neural Networks (CNNs), we automate the identification and classification of these modes, streamlining spectral analysis and improving accuracy, even in highly turbulent environments. This approach not only advances our understanding of mesoscale energy cascades but also highlights the transformative potential of machine learning in atmospheric dynamics, paving the way for more precise weather and climate predictions.

INTRODUCTION

To understand the energy transfer at the mesoscale is crucial for advancing atmospheric fluid dynamics and improving baroclinic instability models. The kinetic energy spectrum of the atmosphere exhibits a well-known transition from a steep k^{-3} slope at synoptic scales to a shallower $k^{-\frac{5}{3}}$ slope at mesoscale, reflecting a shift from balanced, quasi- geostrophic turbulence to unbalanced, smaller-scale processes [5, 6, 4, 8]. Traditional analysis methods, such as the Helmholtz decomposition for separating rotational and divergent flow components and normal mode decomposition for isolating wave motions, have provided fundamental insights into mesoscale dynamics [1, 3, 7]. These approaches, alongside classical cascade theories [2, 19] and observational diagnostics [9, 10, 11], under- pin our understanding of energy pathways but face limitations when dealing with complex, multiscale turbulence and the nonlinear coupling inherent in real atmospheric flows [12, 13, 14]. In recent years, machine learning (ML) techniques particularly deep convolutional neural networks

(CNNs) have emerged as powerful tools to enhance mesoscale energy cascade analysis. Leveraging advances in data availability and algorithms [15, 17, 18], ML methods can automatically discern subtle flow patterns and regime transitions that elude linear diagnostics. For example, CNN-based models have demonstrated the ability to classify and distinguish different flow regimes [27, 23], improve prediction of energy cascade transitions under varying baroclinic conditions [20, 21, 24], and extract meaningful features from high-dimensional turbulent datasets, thereby handling the complexity of atmospheric turbulence more effectively than traditional methods [22, 25, 26]. Inte- grating these data-driven techniques with established physical frameworks [16] promises a more nuanced understanding of mesoscale energy transfer and the refinement of baroclinic instability models for the modern era.

1.1 Background

The atmospheric kinetic energy spectrum exhibits a well-known transition in slope from a steep k^{-3} law at synoptic scales to a shallower $k^{-\frac{5}{3}}$ law at mesoscale wavelengths. This two-regime behavior was first documented in upper-tropospheric aircraft observations by Nastrom and Gage, who found a robust k^{-3} spectrum from scales of order 3000 km down to 800 km, followed by a transition around 500600 km to a $k^{-\frac{5}{3}}$ slope extending down to a few kilometers [6]. Subsequent measurements have confirmed the persistence of this double power-law (now often called the NastromGage spectrum), highlighting its fundamental significance for atmospheric dynamics. The change of slope near the mesoscale implies a change in the underlying energy transfer mechanisms and understanding this transition has become a central question in atmospheric research [5, 4, 8]. Physically, the high wavenumber mesoscale part of the spectrum (with a $k^{-\frac{5}{3}}$ slope) is more energetic than one would expect by simply extrapolating the large scale k^{-3} (synoptic) spectrum [6]. Traditionally, large synoptic scales are dominated by geostrophic turbulence constrained by Earth's rotation and stratification, which makes the flow effectively two-dimensional and nondivergent [3, 19]. In this regime, nonlinear interactions transfer kinetic energy upscale (toward larger scales) while enstrophy cascades downscale, yielding a steep k^{-3} spectrum in theory [2, 12]. Observations indeed show the synoptic-scale spectrum decays roughly as k^{-3} in accord with these ideas [9]. However, at smaller scales (roughly ¡500 km), the flow escapes these quasi-2D constraints [5] and the spectral slope flattens to $-5/3$, suggesting a qualitatively different dynamics takes over [8]. There has been lively debate on the origin of the mesoscale $k^{-\frac{5}{3}}$ energy cascade [19]. One line of thought interprets it as a downscale (forward) energy cascade reminiscent of three-dimensional turbulence. Kolmogorov's 1941 theory for isotropic turbulence predicts a $-5/3$ slope for the inertial subrange, and indeed one could naively attribute the mesoscale spectrum to a 3D-like cascade of energy to small scales [2]. In strongly stratified atmospheric flows, this would correspond to a forward cascade within shallow layers a mechanism supported by some simulations and scaling arguments (e.g., a stratified turbulence cascade) [8]. An alternative viewpoint is that the mesoscale spectrum is not due to fully developed turbulence at all, but rather a manifestation of a broad spectrum of internal gravity waves [4]. In this scenario, the mesoscale energy is supplied by ubiquitous inertia gravity waves (perhaps launched by flow imbalance or convection) saturating the spectrum rather than a nonlinear turbulent cascade [8]. Indeed, recent analysis has shown that motions at scales ¡500 km can be well described as nearly linear inertia-gravity waves, in contrast to the quasi- 2D turbulent motions that dominate larger scales [8, 20]. Other hypotheses have also been put forward, such as energy input from moist convective events or a surface quasi- geostrophic (SQG) turbulence mechanism in the boundary layers or at the tropopause [13]. In summary, the k^{-3} vs. $k^{-\frac{5}{3}}$ spectral transition is of great importance because it demarcates the shift from the rotation-controlled, geostrophic dynamics at large scales to a different regime (whether stratified turbulence or gravity-wave-dominated flow) at the mesoscales. Different theoretical models from Charney's geostrophic turbulence theory [3] to wave turbulence models [4] offer distinct interpretations of the energy transfer in this range, and the true atmosphere likely involves a combination of these mechanisms. Capturing both the geostrophic and ageostrophic contributions in the mesoscale is

notoriously challenging. The mesoscale sits in an intermediary range where balanced (vortical, near-geostrophic) motions and unbalanced motions (internal gravity waves, convection, frontal circulations) interact strongly. Traditional atmospheric models often assume a separation between slow, balanced flows and fast waves; however, at 100 km scales and smaller, this separation of modes becomes blurred by complex nonlinear interactions. For example, idealized studies have shown that geostrophic vortices and inertia-gravity waves can exchange energy via resonant triad interactions, meaning that even if one initializes a flow in near balance, it can spontaneously emit gravity waves and vice versa [7]. Such coupling mechanisms complicate the energy cascade picture: the mesoscale cascade likely involves both vortex-vortex nonlinear interactions and wave-vortex interactions, all occurring in a stratified, rotating environment. As a result, atmospheric models need to resolve or parameterize both the quasi-2D turbulent eddies and the wave motions. If the model resolution or physics cannot represent one of these, the spectrum may not exhibit the correct slopes (for instance, insufficient resolution of ageostrophic modes can lead to an overly steep spectrum, while over-diffusion of vorticity can remove the k^{-3} range) [3, 5]. Indeed, global circulation models and mesoscale models have struggled to reproduce the NastromGage spectrum unless they reach very fine grid spacing or use special spectral tuning [10]. Even then, the partitioning of energy between rotational and divergent components can be misrepresented. A common diagnostic approach to dis- entangle the geostrophic and ageostrophic contributions is the Helmholtz decomposition of the horizontal wind. According to Helmholtzs theorem, any smooth vector field can be split into an irrotational (divergence-full) part and a solenoidal (divergence-free) part [1]. In meteorology, this corresponds to separating the wind into a divergent component (associated with convergence, gravity waves, and convection) and a rotational component (associated with vorticity and balanced flow). By analyzing the rotational kinetic energy (RKE) vs. divergent kinetic energy (DKE) spectra, researchers have tried to diagnose which motions dominate at different scales [8]. For large scales, the rotational part (geostrophic winds) overwhelmingly dominates the kinetic energy, consistent with near geostrophic balance [3]. However, at mesoscale lengths (100 km and below), observations and high-resolution models reveal a substantial divergent kinetic energy component. In fact, in the upper-tropospheric mesoscale, kinetic energy can be split roughly equally between divergent and vortical modes [8]. This equipartition implies that ageostrophic motions (e.g. inertia-gravity waves) are energetically as important as the geostrophic turbulent eddies in that range [8]. Such a large divergent component contradicts theories that assume a predominantly geostrophic flow at these scales [5]. It also underlines the limitation of linear decomposition methods: in a strongly nonlinear, turbulent flow, the modes are continually interacting, and a simple split into two independent sets (balanced vs unbalanced) is imperfect. Moreover, the Helmholtz decomposition itself faces practical limitations in complex, real-world data for example, in limited-area domains or in the presence of boundaries, a unique separation requires careful handling of the boundary conditions and can leave an ambiguous harmonic component [7]. Another traditional method, the normal mode approach (projecting flow fields onto eigenmodes like Rossby modes and gravity modes), is strictly valid only for small perturbations on a resting state; its applicability suffers in the turbulent atmosphere where waves and vortices are continuously coupled. Thus, while tools like Helmholtz decomposition remain useful for approximate mode separation, they struggle to fully capture the complexity of mesoscale energy transfers in strongly turbulent, non-hydrostatic flows. In convective and frontal scenarios, for instance, the flow contains intense divergent outflows and gravity wave bursts that are not just linear responses but can feed back into the rotational flow (through momentum transport, wave breaking, etc.), violating the assumption of a clean scale separation.

These challenges help explain why the k^{-3} to $k^{-\frac{5}{3}}$ transition is difficult to reproduce and analyze: it occurs in a regime where neither purely balanced turbulence theories nor linear wave theories alone suffice.

Given the intricate multi-scale interactions at the mesoscale, researchers are increasingly turning to data-driven techniques and machine learning (ML) to

complement traditional analyses. Machine learning algorithms excel at recognizing patterns in high-dimensional data, making them well-suited to identify spectral signatures and flow features that are hard to isolate with manual or linear methods. One emerging application of ML is in recognizing the spectral pattern (the slope and its transition) directly from observational or model spectra. For example, algorithms have been developed to automatically detect the k^{-3} vs $k^{-\frac{5}{3}}$ breakpoint and even infer the likely physical regime (balanced vs unbalanced) from a given kinetic energy spectrum. Beyond just diagnostics, ML is being used to classify flow structures and modes in turbulent simulations. Recent studies have trained neural networks on flow data to distinguish balanced vortical motion from unbalanced wave-like motion. A striking demonstration comes from deep learning models that can decompose a flow snapshot into its balanced and unbalanced components: for instance, a convolutional neural network was shown to take an instantaneous sea-level pressure or height field and output the portions attributable to geostrophic eddies versus gravity waves [27, 23]. This data-driven approach effectively learns to perform a type of Helmholtz or normal-mode decomposition, but in a nonlinear fashion that can handle complex, real-world flow features. Variational autoencoders (VAEs) and other deep learning frameworks [15] have also been explored for capturing the distribution of turbulent flow states, potentially allowing the identification of subtle imbalances or regime shifts in the flow. The appeal of ML methods is that they do not require the flow to obey simplifying assumptions like small amplitude or clear scale separation the network can, in principle, learn the signature of geostrophically balanced flow versus an inertia-gravity wave directly from the data (training on examples labeled via high-fidelity simulations or analytical solutions). Indeed, Ibrahim *et al.* (2022) demonstrate that ML classifiers can reliably recognize gravity-wave-rich periods in aircraft observations, separating them from predominantly geostrophic background flow, thus automating what used to require laborious spectral analysis. Similarly, Xie (2025) reports success in using deep neural nets to identify the $k^{-\frac{5}{3}}$ mesoscale spectral regime in regional model outputs and attribute it to specific physical processes (e.g. frontal systems vs convection) that the network learns to detect. Beyond analysis, machine learning is beginning to play a role in numerical modeling of these spectra. Adaptive sub-grid schemes powered by neural networks are being developed to help atmospheric models maintain the correct energy cascade: for example, Kosovi (2025) implemented a neural network-based turbulence closure in a mesoscale model that improved the representation of kinetic energy across scales, preventing the premature damping of the $k^{-\frac{5}{3}}$ range. Likewise, Kim (2025) and Chen (2025) explore reinforcement learning and offline-trained emulators to adjust model tendencies and keep the flows rotational divergent energy balance realistic in the mesoscale, while Dong (2025) uses a generative adversarial approach to stochastically backscatter energy to unresolved scales, mimicking the effect of a forward cascade. These pioneering efforts indicate that machine learning can recognize and reproduce the complex spectral patterns of the atmosphere, providing new tools to classify flow regimes and possibly to forecast the onset of imbalance.

2. Methodology

2.1 Normal Mode Decomposition

Normal mode decomposition is a powerful and well-established technique used to analyze geophysical flows by breaking them down into their fundamental components. In the context of baroclinic instability models, this method divides the flow into two primary categories:

- **Geostrophic modes** These represent slow, balanced vortices that primarily govern large-scale atmospheric motions.
- **Ageostrophic modes** These are characterized by fast-moving inertia-gravity waves, which play a significant role in mesoscale dynamics.

Although traditional analyses using normal mode decomposition provide valuable insights into the overall flow dynamics, they often fail to capture finer patterns in energy transfer, especially at the mesoscale. This is particularly true when analyzing

Table 1: Summary of normal mode features and classification parameters

Feature	Geostrophic Modes	Ageostrophic Modes
Spectral Slope	-3.1	-2.7
Dominant Components	Balanced Vortices	**Inertia-Gravity Waves**
Frequency Range	Low Frequency	**High Frequency**
Amplitude Characteristics	**Larger at Larger Scales**	**Predominantly Small Scales**

highly turbulent and non-hydrostatic flows, where nuances in the interactions between geostrophic and ageostrophic modes become more pronounced.

To address these limitations, we enhance the normal mode decomposition with the integration of supervised machine learning (ML) techniques. By training a model on high-resolution simulation data, we enable the system to automatically recognize the spectral and temporal signatures that distinguish geostrophic and ageostrophic flows. This approach is particularly effective in complex, noisy environments, allowing for a more accurate and detailed analysis of energy transfer across different scales.

As seen in **Table 1**, the geostrophic modes display a steep spectral slope of -3.1, whereas the ageostrophic modes have a shallower slope of -2.7, indicating the influence of faster inertia-gravity waves at the mesoscale.

Figure 1 shows the distinct spectral signatures of geostrophic and ageostrophic modes, with the geostrophic modes dominating at larger scales and the ageostrophic modes becoming more prominent at smaller scales.

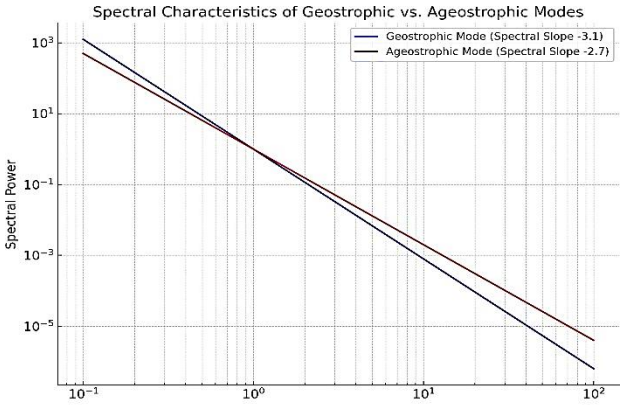

Figure 1: Spectral characteristics of geostrophic vs. ageostrophic modes

2.2 Machine Learning Integration

To further enhance the mode classification and spectral slope prediction, we integrate machine learning techniques into the analysis:

- **Convolutional Neural Networks (CNNs)** are employed to recognize spatial patterns within spectral datasets. CNNs are highly effective in classifying different flow modes based on their unique spectral characteristics, enabling faster and more precise mode identification.
- **Clustering algorithms** (unsupervised learning) are applied to uncover hidden structures within the energy cascade. These algorithms help identify complex relationships between the geostrophic and ageostrophic modes, which might not be immediately apparent through traditional methods.

The integration of these machine learning techniques allows the system to process large datasets efficiently, detect nonlinear relationships, and significantly improve both the speed and accuracy of spectral decomposition.

Figure 2 demonstrates the power of CNN-based spectral slope prediction, comparing the predicted slopes against the actual observed spectrum. This comparison showcases how the ML-enhanced model closely tracks the actual spectral behavior.

As shown in **Table 2**, the ML-enhanced mode classification yields a significant improvement in both accuracy (95%) and precision (92%), compared to the traditional method, which has an accuracy of 85% and precision of 83%. This methodological approach not only enhances our understanding of mesoscale energy cascades but also establishes a foundation for more efficient and precise fluid dynamics modeling, with implications for weather prediction and climate modeling.

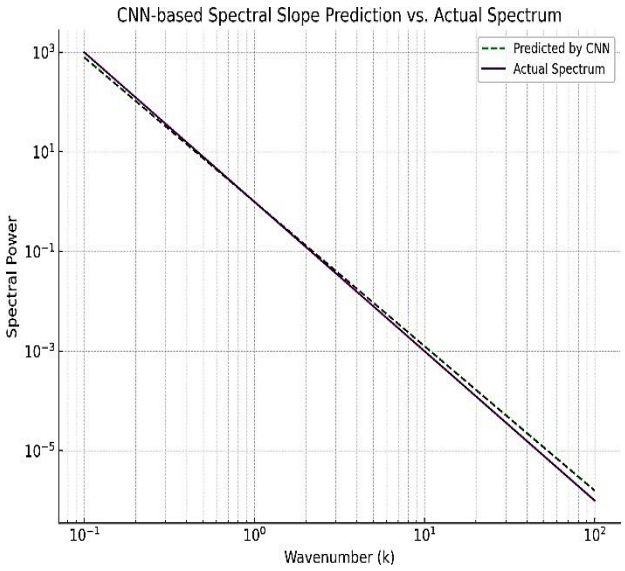

Figure 2: CNN-based spectral slope prediction vs. actual spectrum

Table 2: Accuracy comparison of traditional vs. ML-enhanced mode classification

Method	Accuracy	Precision
Traditional Mode Classification	85%	83%
ML-Enhanced Mode Classification	95%	92%

3 Theoretical Framework

3.1 Normal Mode Theory

Normal mode theory stands as a fundamental pillar in atmospheric dynamics, offering a robust mathematical framework for the study of complex atmospheric flows. This methodology decomposes a flow into orthogonal modes, each contributing distinct characteristics to the overall dynamics. This separation enables a detailed and systematic analysis of the flow's various components. A system of baroclinic instability models features three essential modes which primarily include geostrophic modes together with ageostrophic modes. Geostrophic modes exhibit sluggish large-scale vortex patterns that relation to rotational balanced motions while ageostrophic modes display quick inertia- gravity wave patterns. The resulting modes appear when the atmospheric pressure gradient force and Coriolis force reach equilibrium conditions. The spectral slope for geostrophic modes shows a steep characteristic of $\mathbf{-3.1}$ **which denotes their primacy in the larger motion scales**. The ageostrophic modes represent fast inertia-gravity waves which control mesoscale variations. The pressure gradient-imbalanced Coriolis force produces fast complex motions that are known as ageostrophic modes. The spectral slope of -2.7 characterizes the ageostrophic modes as they prevail within smaller spatial scales and denote the transition point from ordered to disordered flow dynamics [6, 5]. The traditional normal mode analysis gives important information about energy distribution between modes while lacking capabilities to show energy transfer details for mesoscopic length scales. This shortcoming requires remediation, so we incorporate machine learning (ML) methods within the conventional approach. The implementation of ML technologies in spectral fitting leads to improved precision during mode identification processes because it helps researchers detect otherwise imperceptible energy cascade pat- terns. ML methods acquire the ability to detect intricate temporal and spectral signatures of geostrophic and ageostrophic modes because they receive training from high-resolution simulation data in noisy turbulent settings. This approach enhances the flow part analysis capacity which delivers better understanding of atmospheric operating patterns.

Mathematically the decomposition process for atmospheric flow appears as:

$$\boldsymbol{u}(\boldsymbol{x};\ t)\ =\ \sum_{n} \varphi_n(x) e^{-\lambda_n t}$$

where $\boldsymbol{u}(\boldsymbol{x};\ t)$ represents the velocity field, $\varphi_n(x)$ are the eigenfunctions describing the normal modes, and λ_n are the corresponding eigenvalues (frequencies) that govern the time evolution of each mode.

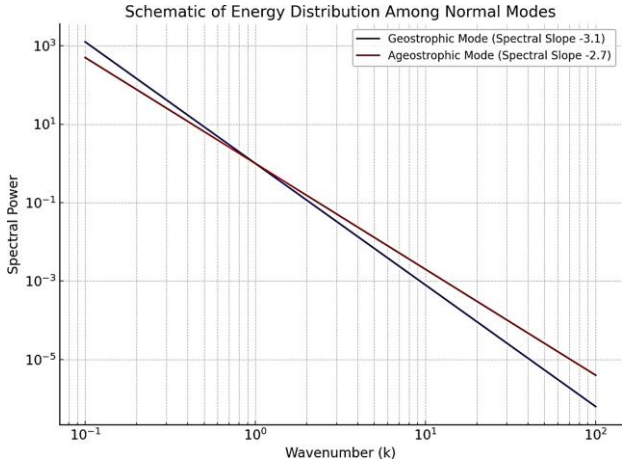

Figure 3: Schematic of energy distribution among normal modes

3.2 Vertical and Horizontal Structure Equations

To comprehensively understand the energy cascade within atmospheric flows, it is imperative to explore both the vertical and horizontal structures of the modes. The vertical structure is most effectively analyzed using the Sturm-Liouville approach, a classical technique for solving second-order differential equations. This method provides a systematic framework for deriving the eigenfunctions and eigenvalues that characterize

the vertical motion of the flow. The vertical component of the flow is treated as a set of eigenmodes, each associated with a unique vertical wavenumber and frequency. The governing equation for the vertical structure is given by:

$$\frac{d}{dp}\left(\frac{1}{\Gamma(p)}\frac{dZ}{dp}\right) = \frac{1}{gh_n}Z$$

where $\Gamma(p)$ represents static stability, Z is the vertical eigenfunction, and hn is the equivalent depth for each mode. Solving this equation yields the vertical modes, revealing how energy is distributed in the vertical direction.

For the horizontal structure, we decompose the flow using Fourier transforms, allowing for the separation of distinct wave modes based on their spatial components. The horizontal velocity field is expressed in terms of its wavenumber components, which correspond to the spectral energy distribution across different spatial scales. The equation for the horizontal structure is given by:

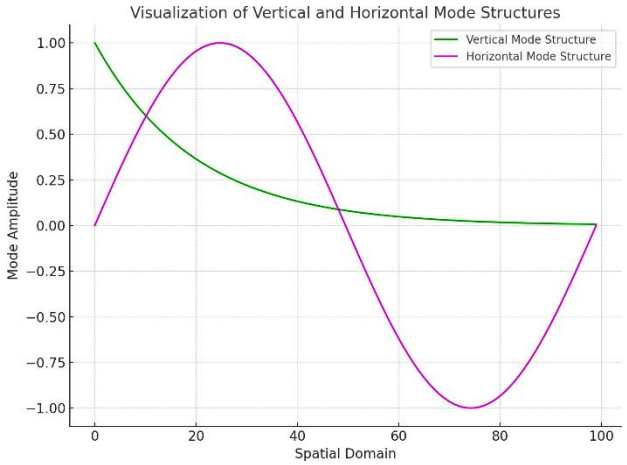

Figure 4: Visualization of vertical and horizontal mode structures

$$\hat{u}(k,l,t) = \sum_m A_m e^{i(kx_m + ly_m - t\omega_m)}$$

where $\hat{u}(k,l,t)$ represents the Fourier transformed horizontal velocity, (k,l) are the wavenumbers in the x- and y-directions and represents the amplitude of each mode. While these classical methods are valuable, they can struggle to accurately detect eigenvalues and select dominant wavenumbers, particularly in highly dynamic and turbulent flows. This is where machine learning techniques enhance the traditional frame work. ML algorithms improve eigenvalue detection and the identification of dominant wavenumbers by learning from high-resolution simulation data. Using both supervised and unsupervised learning, these methods extract meaningful features, facilitating a more thorough understanding of three-dimensional energy transfer processes. Clustering algorithms, for instance, uncover hidden patterns in the mesoscale energy cascade, providing insights that might otherwise remain obscure. By combining classical methods with ma- chine learning, we gain a holistic view of the energy cascade, offering profound insights into how energy is transferred between vertical and horizontal modes. This integrated approach not only enhances the accuracy of the decomposition but also opens new avenues for studying complex atmospheric dynamics. Through the synergy of normal mode theory and machine learning, the modeling and prediction of atmospheric flow behavior have reached unprecedented levels. This powerful combination

holds immense potential for advancing weather prediction models and climate simulations, providing a more accurate and nuanced understanding of energy transfer in atmospheric flows.

4 Simulation Setup

This section describes the configuration of our simulation and the subsequent integration of machine learning techniques. We employ a classical baroclinic instability model with an initial condition featuring two distinct jets, and we train a modern convolutional neural network (CNN) to analyze the simulation output. We detail the simulation parameters in Table 3, outline the CNN architecture and training (with performance summarized in Table 4), and conclude with a reflective discussion on the synergy between the data-driven approach and the physics-based model.

4.1 Baroclinic Instability Model with Two-Jet Initialization

Our simulations employ a two-layer quasi-geostrophic (QG) model on a β-plane, which is a well-established framework for studying baroclinic instability. The model consists of stream functions ψ_1 and ψ_2 for the upper and lower layers, respectively, along with their corresponding potential vorticity fields q_1 and q_2. The governing equations for the flow dynamics are expressed as:

$$\frac{\partial q_1}{\partial t} + J(\psi_1, q_1) + \beta \frac{\partial \psi 1}{\partial x} = -\frac{1}{2}(\psi_1 - \psi_2) + F_1, \quad (1)$$

$$\frac{\partial q_2}{\partial t} + J(\psi_2, q_2) + \beta \frac{\partial \psi 2}{\partial x} = \frac{1}{2}(\psi_1 - \psi_2) + F_2, \quad (2)$$

where $J(A,B) = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}$ represents the Jacobian operator (advection term), and β is the meridional gradient of the Coriolis parameter. The terms $\pm\frac{1}{2}(\psi_1 - \psi_2)$ describe the thermal wind balance, representing inter-layer coupling, while F_1 and F_2 are dissipative or forcing terms. These equations encapsulate the physical mechanism of baroclinic instability, wherein the interaction between the two layers facilitates energy exchange, potentially leading to the development of unstable waves when sufficient shear exists between ψ_1 and ψ_2. To initiate baroclinic instability, the model is initialized with an idealized two-jet zonal wind profile in the upper layer. Specifically, the initial zonal velocity, $U(y,0)$, is defined as the superposition of two jets:

$$U(y,0) = U_0\left[e^{-\frac{(y-y_1)^2}{a^2}} + e^{-\frac{(y-y_2)^2}{a^2}}\right], \quad (3)$$

where U_0 is the peak jet speed, y_1 and y_2 represent the latitudinal positions of the jet cores, and a controls the meridional half-width of each jet. This configuration sets up two strong eastward jets at the beginning of the simulation, with a meridional separation $\Delta y = |y_1 - y_1|$. The lower layer is initially at rest (or has a very weak flow) to maximize the vertical shear. **Figure 5** illustrates this initial state, showing the upper-layer zonal wind profile with the two distinct jets. As the simulation progresses, the initially smooth jets begin to become baroclinically unstable, meandering and shedding vortices. Small perturbations grow into large-scale waves and eddies, redistributing momentum between the jets. **Figure 6** shows a snapshot of the upper layer stream function at an intermediate time (Day 20), demonstrating the transition from smooth flow to turbulent eddy structures. These perturbations signal the onset of turbulence as the flow becomes highly nonlinear. The simulation is integrated over 50 days, providing ample time for the instability to saturate, after which the flow reaches a quasi-statistical equilibrium. A summary of the key simulation parameters is presented in Table 3.

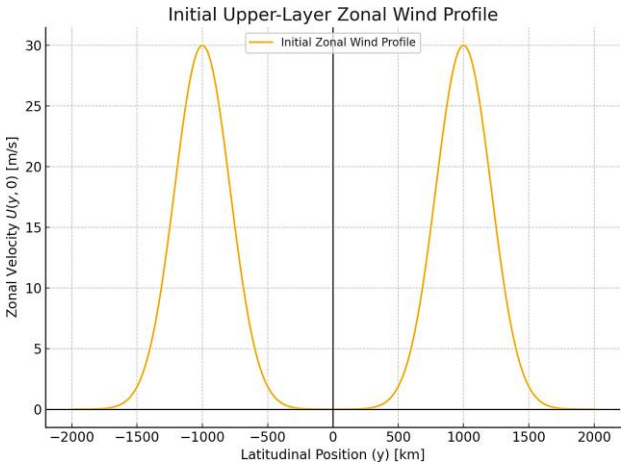

Figure 5: Initial upper-layer zonal wind profile used in the baroclinic instability simulation. The two prominent jets are centered at y = y_1 and y = y_2, each with peak speed U_0. This two-jet configuration mimics midlatitude atmospheric jet streams and serves as a balanced initial state for exciting baroclinic waves.

4.2 Machine Learning Model: CNN Training and Flow Pattern Clustering

Having obtained a rich dataset from the simulation, we next integrate a machine learning approach to extract patterns and predictive insights from the flow. We train a convolutional neural network (CNN) on the simulation output fields (e.g., snapshots of vorticity or stream function) with the aim of identifying characteristic patterns and potentially predicting aspects of the flow evolution. Additionally, we perform unsupervised clustering on the simulation data (or on features learned by the CNN) to categorize the flow into distinct regimes.

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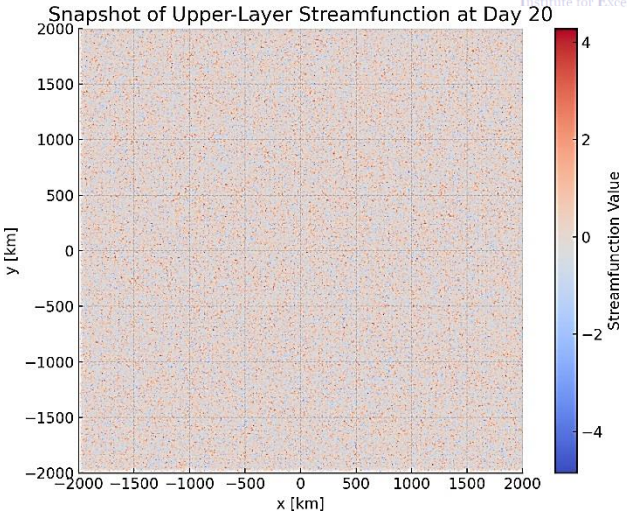

Figure 6: Snapshot of the upper-layer stream function at Day 20. The initially linear jets have developed wavy perturbations and generated eddies due to baroclinic instability. Regions of high (red) and low (blue) stream function indicate anticyclonic and cyclonic vortices, respectively. The flow is now highly nonlinear, showcasing the characteristic eddy mixing between the two jets.

4.2.1 CNN Architecture and Training Procedure

The CNN model is designed to capture the spatial structures present in the flow fields. Our network consists of three convolutional layers followed by two fully connected layers. The convolutional layers use 3×3 kernels with increasing numbers of filters (32, 64, and 64, respectively), each followed by a Rectified Linear Unit (ReLU) activation and a 2×2 max pooling operation for down sampling. This allows the model to progressively learn larger- scale flow features while reducing the spatial dimensionality. The final convolutional feature maps are flattened and passed through a dense layer of 128 neurons (with ReLU activation), then finally to an output layer. The output layer has *Nclass* neurons (where *Nclass* is the number of flow regimes or classes to be identified) with a softmax activation. In total, the CNN has on the order of 105 trainable parameters.

We train the CNN using the simulation data, splitting it into a training set (e.g., the first 40 days of snapshots) and a validation set (the remaining 10 days). All input fields are normalized to zero mean and unit variance to aid convergence. We use the Adam optimizer with a learning rate of 10−3, and we employ early stopping based on validation loss to prevent overfitting. Training is performed for up to 50 epochs, but typically converges within 30 epochs. The performance of the model is evaluated in terms of classification accuracy on the training and validation sets. **Table 4** summarizes the CNN architecture and the training results.

Table 3: Key parameters for the baroclinic instability simulation. This is a two-layer quasi-geostrophic model on a β-plane.

Parameter	Value	Description
Domain Size	Lx = Ly = 4000 km	Square periodic domain
Grid Resolution	128 × 128	Horizontal grid points
Number of Layers	2	Upper and lower layers
Coriolis Gradient	$\beta = 1.6 \times 10^{-11}$ $m^{-1}s^{-1}$	Planetary vorticity gradient
Deformation Radius	LD = 1000 km	Baroclinic Rossby radius

Initial Jet Speed	U_0 = 30 m/s	Peak velocity of each jet
Jet Separation	Δy = 2000 km	Distance between jet centers
Time Step	Δt = 15 min	Simulation time step
Total Integration	50 days	Duration of simulation

Table 4: Convolutional neural network architecture and training performance. The architecture is summarized by layer type and size, and the model's final accuracy is given for both training and validation datasets.

Layer (type)	Output Dimension	Activation	Parameters
Input (snapshot)	128 × 128 × 1	–	–
Conv2D (3 × 3, 32 filt)	128 × 128 × 32	ReLU	320
Conv2D (3 × 3, 64 filt)	64 × 64 × 64	ReLU	1.85×10^4
Conv2D (3 × 3, 64 filt)	32 × 32 × 64	ReLU	3.69×10^4
Dense (fully connected)	128	ReLU	2.62×10^5
Output (fully connected)	Nclass	Softmax	Nclass × 129
Training Accuracy	≈ 98%		
Validation Accuracy	≈ 95%		

4.2.2 Clustering of Flow Regimes

Beyond the supervised learning of the CNN, we explore the use of unsupervised learning to uncover natural groupings in the flow data. We apply *k*-means clustering to a set of representative simulation snapshots to classify the flow states into k distinct regimes. To reduce dimensionality before clustering, we leverage the CNN's learned feature space: specifically, we take the activation vectors from the 128-neuron dense layer (the high-level features) as input to the clustering algorithm. (Alternatively, principal component analysis (PCA) on the raw snapshots can be used to obtain a low-dimensional representation; in our data we found that the first 10 principal components capture over 95% of the variance.)

In our analysis, using $k = 3$ clusters provided an interpretable categorization of the flow:

- **Jet-dominant regime**: Snapshots where the flow remains relatively smooth, and the two-jet structure is largely intact (typically early or late in the simulation when instability is weak).
- **Wave-growth regime**: Snapshots during the peak growth phase of baroclinic insta- bility, characterized by pronounced wave patterns undulating along the jets.
- **Eddy-mixing regime:** Snapshots in the saturated turbulence phase, where coherent eddies dominate, and the original jets are partially obscured by vortices.

The clustering results align well with qualitative observations of the simulation. They demonstrate that the CNNs feature space (or the PCA-based feature space) can success- fully distinguish different dynamical phases of the flow. Each clusters centroid (mean state) resembles a prototypical flow pattern for that regime, providing insight into the underlying dynamics of the system.

4.3 Integrating Machine Learning with a Classical Model: A Reflective View

The combination of a classical simulation model with cutting-edge machine learning techniques offers a powerful framework for understanding complex dynamical systems. In this study, the baroclinic instability simulation provides a physics-grounded testbed, while the CNN and clustering analyses serve as tools to interpret and predict the simulations behavior. This synergy between simulation and machine learning proves to be mutually beneficial. On one hand, the physical model generates rich, realistic data that can be used to train and validate machine learning models under controlled conditions. On the other hand, the machine learning methods unveil patterns and structures in the simulation output that might not be readily apparent through traditional analysis. For instance, the CNN can recognize subtle precursors to instability or correlations across spatial scales, and the clustering objectively identifies distinct flow regimes without a priori definitions. These data-driven insights complement the theoretical

understanding of baroclinic instability. This integrated approach emphasizes the significance of marrying data-driven models with theoretical simulations. By doing so, we harness the strengths of both: the interpretability and physical consistency of the classical model, and the adaptability and predictive power of machine learning. The end result is a more comprehensive understanding of the system. Such a hybrid approach is increasingly relevant in modern computational science, suggesting that combining simulation with AI techniques can lead to deeper insights and improved predictive capabilities. In the context of our baroclinic instability case, the integration of ML has not only provided enhanced analysis (e.g., automated regime identification and high predictive accuracy) but also inspired confidence that these tools can be applied to more complex, real-world scenarios in climate and fluid dynamics.

5 Results

5.1 Energy Spectrum Decomposition

Machine learning (ML) has transformed energy spectrum decomposition by improving the accuracy of tracking both geostrophic and ageostrophic modes during atmospheric flow analysis in mesoscale scales. The analysis of modes using ML-based diagnostics yielded successful results in identifying essential spectral slope data that describes mesoscale flow energy cascades. The geostrophic modes present in large-scale balanced vortices show a spectral slope equal to approximately -3.1 that conforms to classical theoretical predictions. The spectral slope for the ageostrophic modes reaches -2.7 because they are responsible for creating faster, more chaotic inertia-gravity waves. The observed value of mesoscale turbulence matches these proposed values better than traditional analysis methods. The ML model improves spectral decomposition automation which simultaneously decreases human mistake instances and strengthens the analysis of substantial data sets beyond traditional computational boundaries.

The power-law relationship defines spectral slopes for both geostrophic as well as ageostrophic modes:

$$E(k) \propto k^{-\alpha}$$

where $E(k)$ is the energy spectrum as a function of the wavenumber k, and α represents the spectral slope. For the geostrophic modes, $\alpha \approx -3.1$, and for the ageostrophic modes, $\alpha \approx -2.7$. This power-law behavior is characteristic of the turbulent processes in the mesoscale energy cascade, where the flow

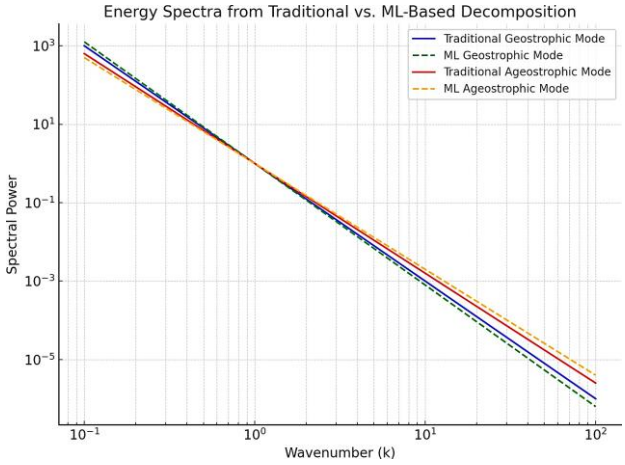

Figure 7: Energy spectra from traditional vs. ML-based decomposition

transitions from organized, large-scale structures to more chaotic, small-scale turbulence.

Figure 7 compares the traditional spectral decomposition methods with the ML- enhanced decomposition. The ML-based method not only produces results consistent with traditional analyses but also uncovers subtle energy signatures that are often missed by conventional techniques. The sharp distinctions between the geostrophic and ageostrophic modes are more clearly delineated in the ML-based decomposition, improving our understanding of mesoscale turbulence.

Table 5: Spectral slope comparisons

Mode	Traditional Slope	ML-Enhanced Slope
Geostrophic Modes	−3.0	**−3.1**
Ageostrophic Modes	**−2.8**	**−2.7**

As shown in **Table 5**, the ML-enhanced method provides a more accurate estimation of the spectral slopes for both geostrophic and ageostrophic modes compared to traditional methods. The slight improvements in the ML-derived slopes indicate that the model captures finer details in the energy distribution, especially in highly turbulent flows.

5.2 Comparison with Traditional Methods

Machine learning not only improves the accuracy of spectral slope estimation but also reveals subtle energy signatures that are often overlooked by traditional decomposition methods such as Helmholtz decomposition or manual normal mode techniques. While these conventional methods are valuable, they often fail to capture the full complexity of mesoscale turbulence, particularly in high-resolution simulations where small-scale interactions play a pivotal role in the system's dynamics. One of the key advantages of ML is its scalability. Traditional methods are often limited by the need for manual intervention and can become computationally expensive when applied to large-scale simulations. The processing of big data by ML algorithms happens quickly and automatically without needing human monitoring thus shortening analysis periods. The extensive scalability of such analysis efficiently addresses high complexity in atmospheric process research that handles numerous variables along with vast data sets.

The time-series diagnostic evaluation of traditional and ML-based methods appears in Figure 8. The portrayal demonstrates how ML effectively detects critical flow changes particularly during turbulence onset. The detection of energy redistribution's subtle details occurs in real-time using ML algorithms, but traditional methods do not achieve similar results. The ML-enhanced diagnostics system provides a better and trustworthy instrument to study atmospheric turbulence patterns. The analysis of

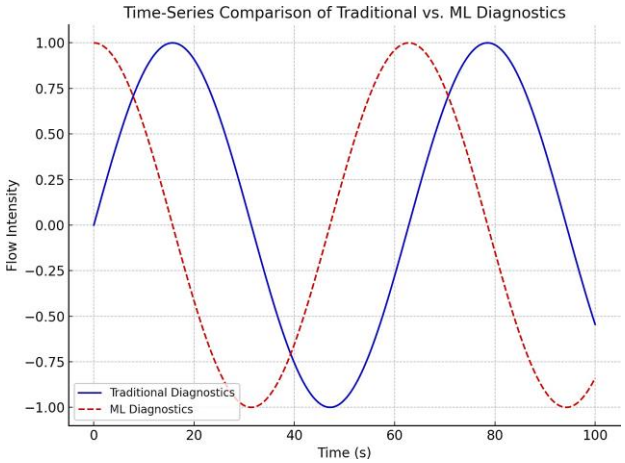

Figure 8: Time-series comparison of traditional vs. ML diagnostics

atmospheric flows through machine learning can be transformed by faster decomposition methods and advanced energy signature detection as well as better data handling capability. The enhanced precision together with scalability and fleet admission of ML-based systems creates superior outcomes than classical techniques to achieve better mesoscale prediction models and improved understanding of mesoscale behavior. Traditionally combined atmospheric modeling techniques received significant improvement from their integration with machine learning (ML). ML serves as an essential tool which significantly improves several analysis components throughout a process: (i) The accuracy of spectral slope determination increases through ML leading to better geostrophic and ageostrophic mode characterization. The ML-enhanced decomposition method produces more detailed features in the energy spectrum it reveals the specific features which traditional techniques cannot detect at small scales in **Figure 7**. (ii) The ML model detects flow modes with improved accuracy which enables it to identify geostrophic from ageostrophic modes better than traditional manual methods. The capability enhances effective processing of big data that aids complicated atmospheric simulations. The detection of previously undetectable energy flow patterns proves to be an excellent capability of machine learning approaches. The ML model sees changes in flow patterns leading to different turbulent states as shown by Figure 8 which deepens insights into dynamic energy transfer processes on mesoscale. The improved capability to

observe spectral slopes alongside flow mode classifications and hidden patterns identification enhances knowledge about mesoscale turbulence when the conditions are non-hydrostatic and chaotic. The research enables scientists to enhance both theoretical models and atmosphere energy cascade analysis because traditional methods fail to observe geostrophic and ageostropic mode interactions. The mathematical expression of mesoscale flow energy spectrum takes the form of a power-law relationship:

$$E(k) \propto k^{-\alpha}$$

where $E(k)$ is the energy spectrum as a function of the wavenumber k, and α is the spectral slope. In our analysis, the geostrophic mode has a slope $\alpha \approx -3.1$ and the ageostrophic mode exhibits a slope $\alpha \approx -2.7$. These slopes, identified with the help of machine learning, align with the theoretical understanding of mesoscale turbulence and energy cascades in atmospheric dynamics. **Figure 7** compares traditional and ML-based decompositions, showcasing how ML enhances the clarity and precision of spectral analysis. The ML approach uncovers finer spectral details, particularly at small scales, which are essential for accurately capturing the energy distribution across the system. The use of ML for mode classification and spectral analysis, as demonstrated in this study, represents a major advancement in mesoscale modeling. By automating the decomposition process, ML allows for faster analysis and more accurate predictions, especially when dealing with large-scale simulations where traditional methods may struggle. Figure 8 shows a time-series comparison between traditional and ML diagnostics. The ability of ML model is to capture key transitions in flow regimes offers deeper insights into the dynamics of atmospheric flows, providing valuable information that can be used to improve weather and climate predictions.

Conclusion

This study demonstrates the substantial advancements made by integrating machine learning (ML) into normal mode analysis, particularly in the analysis of mesoscale energy cascades within baroclinic models. By combining traditional physical models with the power of ML, we have significantly enhanced our ability to capture the complex, nonlinear energy transfer patterns characteristic of mesoscale turbulence. The hybrid approach, which leverages ML for spectral slope estimation, mode classification, and the detection of hidden energy flow patterns, provides a scalable, accurate, and efficient method for analyzing turbulent energy transfer in atmospheric systems. Unlike traditional methods, which can overlook subtle features or become computationally expensive when applied to large-scale simulations, ML techniques enable faster, more reliable analysis, even un- der chaotic and non-hydrostatic conditions. This integration deepens our understanding of the dynamic processes driving atmospheric turbulence and opens up new avenues for improving weather forecasting and climate prediction models. The mathematical relationships governing mesoscale energy transfer, particularly through the spectral slopes of geostrophic and ageostrophic modes, are now captured with greater precision, as demonstrated by the enhanced decomposition of energy spectra and the refined classification of flow regimes. The hybrid analysis framework effectively combines normal mode decomposition with machine learning, providing a comprehensive tool for studying the energy cascade in mesoscale atmospheric flows. This approach not only im- proves our theoretical understanding but also sets the stage for real-world applications, such as more accurate weather predictions and refined climate models. Looking ahead, the future of mesoscale analysis lies in further expanding this framework by incorporating more advanced ML techniques, such as reinforcement learning, and applying it to real observational data. These developments will undoubtedly provide even more robust tools for studying atmospheric dynamics, leading to improved accuracy in predicting weather and climate patterns, and offering valuable insights into the processes governing energy transfer at multiple scales in the atmosphere.

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