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# NUMERICAL ANALYSIS AND ENVIRONMENTAL IMPACT OF AFTERBURNING EXHAUST PLUMES IN ROCKET AND JET ENGINES

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**Abstract**

*Rocket exhaust plume dynamics are essential to aerospace propulsion, influencing performance, safety, and environmental impact. After combustion reactions take place because of leftover oxygen in the plume, there is a significant increase in radiation intensity and exhaust gas temperature. This study is intended to examine and assess the improvements in computational modeling and diagnostic methods to comprehend rocket exhaust plume behavior. The focus is on enhancing predictive capabilities, optimizing propulsion system performance, and reducing environmental and operational issues related to plume dynamics. The investigation of rocket exhaust plumes has progressed significantly due to the emergence of high-fidelity Computational Fluid Dynamics approaches like Large Eddy Simulation and Unsteady Reynolds-Averaged Navier-Stokes. These models better the precision of plume flow forecasts by accurately representing turbulence, heat transfer, and intricate chemical interactions. The combination of computational models with cutting-edge diagnostic instruments, especially laser-based methods, has increased the validation of simulation outcomes and refined propulsion system designs. Nonetheless, challenges persist in faithfully depicting extreme thermodynamic situations, evaluating plume-induced structural impacts, and tackling environmental challenges. In addition, research is necessary to advance simulation frameworks, enhance experimental validation, and create sustainable propulsion approaches. Developing these areas will be necessary for designing future aerospace propulsion systems that are both efficient and naturally sustainable.*

## INTRODUCTION

Hot gases are produced in the combustion chamber of rocket motors and these gases are expelled through the nozzle, creating thrust by interacting with the air. In the mixing layer, the after burning reaction occurs due to the presence of certain oxygen particles incorporated into the plume; thus,

Radiation intensity and temperature of exhaust gases are increasing significantly. Therefore, the exhaust emissions from rocket engines exhibit elevated thermal conditions, significant velocities, and substantial radiation levels. For these reasons, the plume temperature is a significant parameter for evaluating rocket engine performance providing insights into internal combustion process and overall engine performance. In current times, solid fuel combustion has become a popular choice for spacecraft propulsion due to its lower manufacture costs, simplicity and safety [1]. Numerically investigated the flow driven instability generated because of surface, corner or obstacle vortex shedding by placing Inhibitors in combustion chamber for controlling burning of fuel grains. Detached Eddy Simulation has applied to combustion chamber containing inhibitor and the nozzle [2][3]. Results showed that all the configurations were similar except for the inhibitor position. Where maximum amplitude of the pressure oscillation has observed when the inhibitor becomes closer to the diameter of combustion chamber. Further numerically investigated the Inhibitor vortex driven stream because of transient combustion with pressure oscillation. Simulations performed as Detached Eddy Simulations (DES) for precisely predicting flow behavior. Simulations performed and compared with available experimental data. Results showed that frequency amplitude increases importantly and generates acute acoustic environment [4][5].

### 1.1 Background and Significance of Afterburning Exhaust:

The invention of the jet engine during the 1930 by Sir Frank Whittle in the UK and Hans von Ohan in Germany laid the foundation for modern high-speed aircraft. Early jet engines, like those used in World War II aircraft, did not feature afterburners [6][7] .The concept of afterburning was introduced in the 1940 as a method to increase thrust, particularly in military aircraft. The German Me 262, the first operational jet fighter, experimented with early forms of afterburning technology Era and Infrared Signature Management (1950–1960) [8][9]. Last of the 1950 U.S. engine designers began to work on advanced military engines which combined extraordinary high-speed capabilities, achieved through high thrust-to-weight ratios and afterburners with the efficiency and lower specific fuel consumption offered by fanjet technology. The P&W TF30, which was tested with a duct-burning turbofan in 1956, burned both fan and turbine exhaust air in the same afterburner. The P&W TF30 was the first operational afterburning turbofan, making it a significant and challenging improvement. Development of the TF30 initiated in 1959 to support what later became the TFX program in 1961. The TFX program, which led to development of the Common Dynamics F111, aimed to create a large supersonic fighter/bomber to fulfill the both Air Force and Navy carrier-borne aircraft requirements [7] . Afterburning, commonly associated with jet engines, involves injecting additional fuel downstream of the primary combustion chamber to achieve a thrust boost. This technique, though not widely adopted in traditional rocketry, has been a topic of research and experimentation for niche applications requiring exceptionally high thrust or dual-mode operation where a rocket may operate in both atmospheric and vacuum environments. During the 1980s, research on supersonic such as the Concorde also focused on the environmental effects of high-altitude emissions from afterburning system [10]. Supersonic and hypersonic aircraft, such as the SR-71 Blackbird and more recently the X-51 Wave rider, pushed the boundaries of afterburning technology. Research focused on understanding the behavior of exhaust plumes under extreme conditions like high temperatures and rapid gas expansion at supersonic and hypersonic speeds [11] . Afterburners became a standard feature in high-performance military jets like the F-86 Sabre and MiG-21, which required extensive research into afterburning dynamics at high speeds [12].

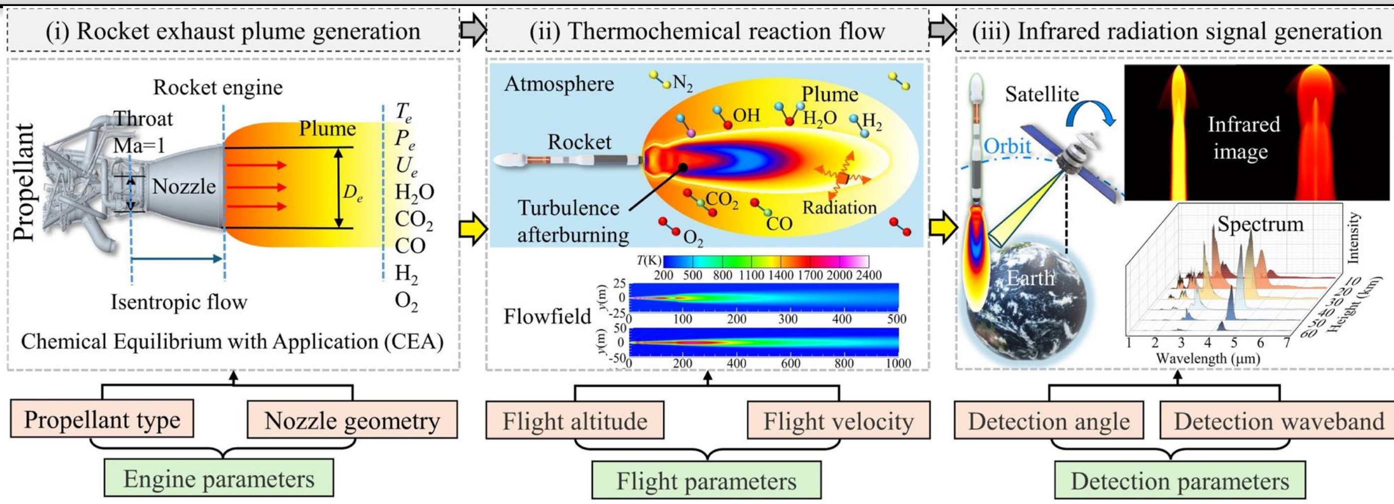


**Fig. 1. Schematic diagram of infrared radiation signal parameters transmission of the rocket engine exhaust plume [13].**

Equilibrium calculations for the proposed NASA Space Shuttle solid-fuel rocket exhaust were performed and parameterized in terms of the oxidizer/fuel ratio [14]. Generating an infrared signal from a rocket engine exhaust plume requires the parameter transfer process shown in Fig 1. the plume flows through the engine's combustion chamber and nozzle, undergoes a thermo-chemical reaction and produces a radiation signal. Several additional components of the parameter transfer process help from the detector that ultimately captures the radiation signal. The infrared radiation characteristics of a rocket engine jet depend on many parameters, including the engine's spectral properties (fuel type, nozzle design, engine thrust), flight condition (velocity, altitude) and detection parameters (angle and wavelength range) [13].

### 1.2 Importance of understanding plume dynamics

The plume refers to the dynamic configuration of elevated-temperature rocket exhaust gases (and, in certain instances, also includes small particles) that emanates from the rocket nozzle. This gaseous formation exhibits a lack of uniformity in terms of structure, velocity, and chemical composition. It encompasses multiple distinct flow regions and supersonic shock waves. Typically, it is discernible as a brilliant flame, radiating substantial energy across the infrared, visible, and ultraviolet wavelengths of the electromagnetic spectrum, and serves as a significant source of acoustic emissions. Many plumes generate a residual trail comprising smoke, vapor, or noxious exhaust gases. At elevated altitudes, a portion of the plume gases may reverse course around the nozzle and interact with various components of the aerospace vehicle [15][16] . Understanding the plume dynamics of rocket motors is essential for optimizing propulsion systems, ensuring safety, mitigating environmental impacts, and achieving mission success. Rocket exhaust plumes, which consist of high-temperature, high-velocity gases and particles, exhibit complex fluid dynamic and chemical behaviors. The performance of rocket engines is closely tied to how effectively exhaust gases are expelled through the nozzle. Plume dynamics affect the distribution of energy within the flow, influencing thrust and specific impulse. Nozzle optimization for staged-combustion engines relies on accurate modeling of plume expansion at different altitudes to maximize thrust [17][18] . Rocket plumes generate intense thermal loads and mechanical stresses, which can impact on the structural components of the vehicle and surrounding infrastructure. The Space Shuttle Main Engine (SSME) displayed the high temperature plume interactions, the use of advanced materials and cooling systems for the nozzle and surrounding structures[19]. Plume dynamics can cause exhaust gases recirculation, contamination of spacecraft surfaces and interference with instruments. Plume contamination during Apollo lunar missions highlighted the need for advanced modeling to forecast and reduce chemical deposition on rockets[20] . Solid rocket motors release aluminum oxide particles that can persist in the stratosphere, impacting radiative forcing and ozone chemistry [21] [22] . For planetary exploration missions, rocket plume dynamics play a crucial role in landing and surface operations. The Apollo lunar landers faced substantial regolith displacement caused by rocket plumes, highlighting the need for surface hazard mitigation strategies. SpaceX's Falcon 9 rocket depends on detailed plume analysis to ensure safe and reliable landings throughout its recovery operations [23].

**1.3. Aim and Objectives:**

The aim of this review is to analyze and evaluate advancements in computational modeling and diagnostic techniques for understanding rocket exhaust plume dynamics. This study focuses on improving predictive capabilities, optimizing propulsion system performance, and mitigating environmental and operational challenges associated with rocket plume behavior.

**The key objectives of this review are:**

1. To explore high-fidelity Computational Fluid Dynamics (CFD) approaches for simulating rocket exhaust plumes, incorporating turbulence modeling, radiative heat transfer and chemical kinetics.
2. To assess the role of advanced numerical techniques such as Large Eddy Simulation (LES) and Unsteady Reynolds-Averaged Navier-Stokes (URANS) in improving the accuracy of plume flow predictions.
3. To evaluate the impact of plume-induced thermal and mechanical loads on rocket components and their implications for propulsion system design and safety.
4. To investigate modern diagnostic tools, including laser-based techniques, for real-time monitoring of plume properties and their contributions to propulsion system optimization.
5. To identify key challenges in computational modeling of rocket exhaust plumes and propose future directions for enhancing simulation accuracy and efficiency.
6. To examine the environmental implications of rocket exhaust plumes and discuss strategies for minimizing their impact on atmospheric conditions and space operations.

**1. Thermodynamic Principles in Exhaust Plume Formation**

In the propulsion systems exhaust plumes are a main part, instead of the final point of energy transfer from combustion to thrust. This operation is governed by Complex thermodynamic process that is transform chemical energy in propellants into kinetic energy, dynamic the motion of rockets and jet engines. Optimizing engine performance is a fundamental principle in reducing the environmental impact and ensuring structural integrity. Heat transfer, rapid expansion and high temperature combustion are interconnected phenomena of the thermodynamic of exhaust plumes. Exhaust gases of propulsion systems consist of combustion products like water vapor($H_2O$), carbon dioxide ($CO_2$) and other species, undergo rapid cooling and pressure changes as they exit the engine nozzle. Temperature, velocity and structure are the plume's physical characteristics when these changes are governed. Fundamental thermodynamic principles behind plume formation include the laws of conservation of mass and energy, gas behavior under varying temperature and pressure situations and the principles of chemical equilibrium. These principles govern the efficiency of energy conversion in propulsion systems and affect plume characteristics, including shock structure, temperature gradients and radiative properties. For example, nozzle design plays a vital role in controlling how exhaust gases increase and accelerate, affecting thrust generation and plume visibility [18]. Meanwhile the thermodynamic performance of exhaust plumes and efficiency of propulsion are important implications for the environment. The high-temperature gases in plumes contribute to the increase of secondary chemical species, such as nitrogen oxides ($NOx$) and soot particles, which can affect air quality and atmospheric chemistry. Understanding the thermodynamic pathways of these reactions is essential for developing cleaner propulsion technologies [24].

**2.1. Combustion in Jet and Rocket Engines**:
Combustion phenomena play crucial character in many industrial applications, including heating systems, propulsion engines and generation of power. Combustion chamber is an area where fuel and air are mixed for the phenomena of combustion [25]. For combustion systems, a combustion chamber is actually the apparatus in which fuels burn at high temperature and pressure large quantity of heat and gases. For which it has to be much strong to sustain these high temperature and pressure values [2]. In propulsion engines such as Solid Rocket Motors these highly pressurized exhaust gases are allowed to expand through nozzles [26]. As these gases expand through nozzle, they accelerated to high speed and pushed the engine in opposite direction.

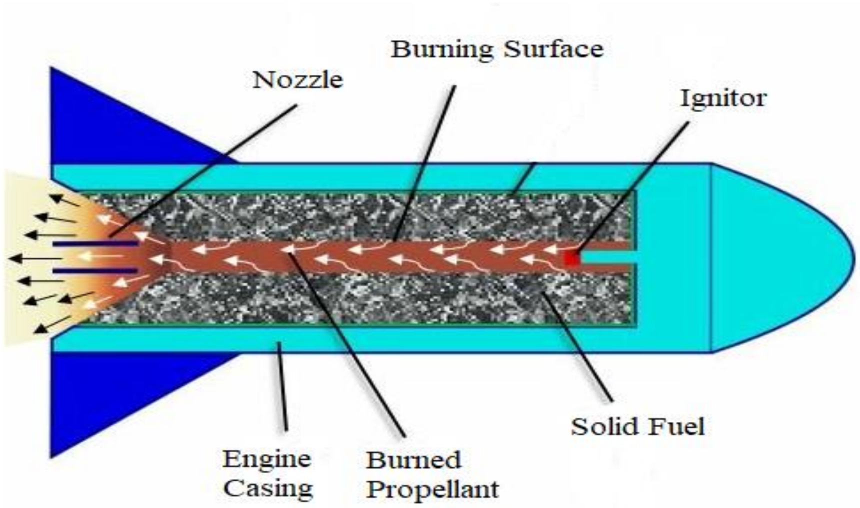


**Fig. 2 Combustion chamber of Solid Rocket Motor [27]**

Combustion chamber for SRM consist of two or more fuel blocks combined by inhibitors where fuel burn slowly and form discontinue structures at junction of fuel segments by producing vortex shedding [4].
Pressure oscillations generated because of vortex shedding interact with chamber acoustics and become cause of excitation of acoustic modes [28].
Pressure oscillations in combustion chambers are activated in engines because of relationship between acoustic standing waves and the combustion where this relationship among vortex shedding and acoustic standing waves always supports oscillations [29]. Combustion in a jet engine involves the chemical reaction between a fuel commonly a refined hydrocarbon like Jet-A or kerosene and atmospheric oxygen. This reaction releases energy in the form of heat and high-pressure gases, which are harnessed to generate thrust. The process takes place in the combustion chamber of the engine, where fuel is sprayed into compressed air supplied by the compressor stage. The air-fuel mixture is ignited, producing hot gases that expand and accelerate through the turbine and nozzle[30]. Jet engine combustion is designed to be continuous, efficient, and stable, even under varying conditions of altitude and speed. Main tasks contain ensuring complete combustion, decreasing pollutants such as nitrogen oxides (NOx), and managing the extreme temperatures inside the engine to prevent material degradation [31].

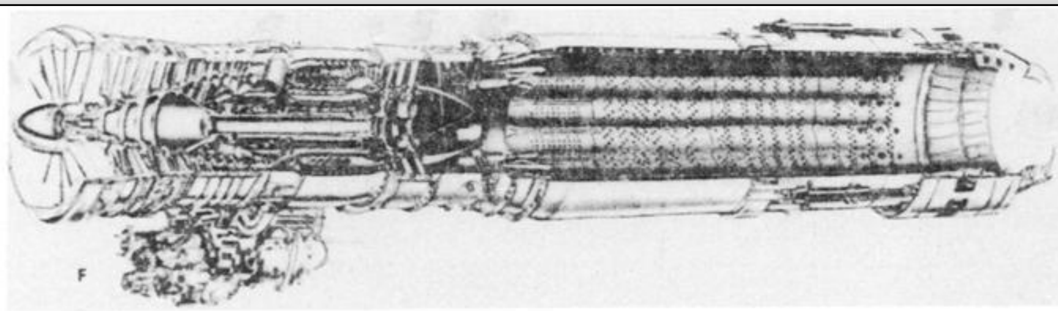

**Fig. 3 Jet engine with afterburner. [31].**

### 2.2. Heat Release and Energy Dissipation in Plumes:

Plumes, which arise in many industrial and natural processes, are considered by the increasing movement of buoyant fluid or gas due to density differences. Heat release and energy dissipation show serious roles in governing the dynamics and performance of plumes, inducing their structure, diffusion and effect on the surrounding environment. Considering these phenomena is vital for applications ranging from environmental pollution control to fire protection and atmospheric studies. Combustion chamber of rocket motor and geothermal process or industries are the main source of heat discharge in plume. This heat induces buoyancy, which subsequently generates turbulent motion, mixing the plume with the ambient fluid. Radiative heat transfer processes and the interplay of convective to determine the thermal characteristics of plume, influencing its spatial extent and vertical rise [32] .Furthermore, energy dissipation occurs as the plume's kinetic energy is convert into heat through viscous forces and turbulence, which impact the stability of plume and entrainment [33]. In environmental applications, including the investigation of pollutant spreading from smokestacks, understanding heat release and energy dissipation is dynamic for forecasting plume increase and conveyance. The energy discharge rate directly affects the smoke production and flame height, they are serious for hazard calculation and mitigation schemes [34]. Experimental and computational techniques have provided deeper insights into these mechanisms. Recent studies have emphasized the importance of turbulence modeling and high-resolution thermal imaging in understanding heat transfer and dissipation in plumes. Exothermic chemical reactions in the combustion chamber and the subsequent reactions in the exhaust are the sources of heat discharge of rocket motors.

These reactions produce the gases at extreme temperature and pressure, which drive the high-speed exhaust through the nozzle. Heat transfer mechanisms, containing radiation, conduction and convection, contribute to the plume's thermal dynamics, affecting its interaction and shape with the atmosphere. Energy dissipation in rocket plumes results from turbulence and mixing as the exhaust gases expand and interact with the ambient atmosphere. The change of kinetic energy into heat through viscous dissipation influences the plume's stability, acoustic characteristics, and the entrainment of atmospheric gases. Accepting these phenomena is dynamic for monitoring acoustic loads on spacecraft, forecasting plume-induced thermal properties, and designing effective plume shielding systems [35]. Exhaust gases of rocket motor frequently leave the nozzle exit at very high temperatures. Heat transfer is required to understand and correctly predict the thermal effects arising from the existence of a hot gas-particle mixture flowing in the vicinity of the essential elements of a rocket. Computation of the heat fluxes emitted rocket exhaust plume directed towards the foundational aircraft is a relatively recent development in the field.

The motivation for this inquiry stems from the necessity for precise quantification of the cumulative thermal load exerted on the rocket base aircraft, which is essential for the accurate dimensioning of its thermal shield. Since thermal radiation is the dominant mode of heat transfer in coal-fired utility boilers. This is the field in which the research about techniques for the solution of the radiative transfer equation has been historically supported. A complete description and comparison of the most main methods is provided in[36]. Monte Carlo method [37] and zone method [38] are used to solve the radiative transfer equation. They are capable of given that exact solution. If the exhaust is slightly over expanded, the barrel shock begins at the nozzle lip, instead of at a certain distance downstream. In both cases, the plume gases flowing through this shock are compressed above the ambient pressure, therefore the method repeats itself to form the shape of Mach diamonds, i.e., an alternative series of shock waves and expansion fans. This can be clearly seen in the liquid rocket engine smoke due to its high transparency, as shown in Fig.4.

**Fig. 4 Liquid rocket engine plume (courtesy: Swiss Propulsion Laboratory) [36]**

**2.3. Chemical Kinetics Governing Afterburning**

The performance and efficiency of rocket propulsion systems are deeply influenced by the chemical kinetics underlying the combustion processes, including the phenomenon of afterburning. Afterburning refers to the continued combustion of unburned propellants, oxidizers, or reaction products in the exhaust stream after leaving the combustion chamber. This secondary combustion process can significantly affect the rocket's thrust, thermal management, and environmental emissions. In rocket motors, particularly in solid and hybrid propulsion systems, incomplete combustion within the combustion chamber often leaves a fraction of the fuel or oxidizer unreacted. These unburned components encounter high-temperature, oxygen-rich conditions in the exhaust plume, initiating additional chemical reactions. The extent and rate of these reactions are governed by principles of chemical kinetics, which command the speed and pathways through which reactants are changed into products.

The emissions generated by jet turbine engines contain a multitude of chemical species containing carbon monoxide($CO$), nitric oxide($NO$) , nitrogen dioxide($NO_2$), and carbon dioxide($CO_2$). Furthermore, unburnt hydrocarbons (UHCs) are present in the exhaust of the engine because of the degradation of complex molecules within the fuel through the combustion process. The identification of UHCs originating from jet engines is imperative for the proof of engine efficacy and the surveillance of environmental pollutants [39].

The differential equations that describe finite rate reaction mechanisms, commonly employed to assess afterburning reaction kinetics within exhaust plumes, exhibit significant stiffness and present significant challenges for integration in the absence of specialized integrators.

The integration system used in Aero Sandbox is not suitable for rigid equation systems and cannot reliably integrate detailed kinetic chemical mechanisms. To make the kinetics of the reaction more computing in aero sandbox, a model of the kinetics of the global mechanism was developed which reduce the order of global mechanism. This model was established to reflect the main physics and behavior restrictions and to find the simplest models of afterburning kinetics after the experience of being performed on a scale. The model of kinetic accepts global reaction with a single reaction rate. Therefore these Completely fuel species are consumed at the similar rate (comparative to their primary quantities in the plume)[19]. The following subsections are defined the more detail.

**2.3.1 Global reaction mechanism**

Suppose total reaction mechanism is

$$X_{CO}CO + X_{H_2}H_2 + X_{OH}OH + X_{O_2}O_2 \rightarrow X_{H_2O}H_2O + X_{CO_2}CO_2 \quad (1)$$

Where $H_i$ are the stoichiometric constants for all speciesi. It is assumed that the global reaction is permanent, (the following coefficient used in [40] for low altitude plumes), and reactions with other species are ignored.

The accurate determination of stoichiometric coefficients $X_i$ is essential to ensure that the reaction mechanism is appropriately balanced. The stoichiometric coefficients affecting the fuel species ($CO, H_2$ and $OH$) are occupied as their corresponding initial mole fractions $wei$ at the end of the nozzle. The$O_2$ stoichiometric coefficients and the resulting product species can be determined based on the coefficient of the function. These coefficients are used to show the fuel species to confirm the mechanism is balanced.

**The following equations are used to determine the coefficients:**

$$X_{CO} = w_{e,CO} \tag{2}$$

$$X_{H_2} = w_{e,H_2} \tag{3}$$

$$X_{OH} = w_{e,OH} \tag{4}$$

$$H_{O_2} = \frac{1}{2}X_{CO} + \frac{1}{2}X_{H_2} - \frac{1}{4}X_{OH} \tag{5}$$

$$X_{H_2O} = X_{H_2} + \frac{1}{2}X_{OH} \tag{6}$$

$$X_{CO_2} = H_{CO} \tag{7}$$

### 2.3.2 Global reaction rate equation

The global reaction rate $r$ (units: molm−3 s−1) is a result of the global reaction mechanism such that these species production rate $w_i$ (units: $g\,m^{-3}s^{-1}$ ) can be defined as

$$w_i = v_i M_i r \tag{8}$$

Where $i$ is species, $M_i$ is molecular weight, $v_i = +X_i$ for product species, and $v_i = -X_i$ for reactant species.

Suppose that $r$ is a global reaction rate which is a form of Arrhenius equation. It is used to model elementary reactions in the detailed form of reaction mechanisms

$$r = Ae^{-E_a/RT}[fuel]^{n_{fuel}}[O_2]^{n_{O_2}}[gas]^{n_{gas}} \tag{9}$$

Here $T$ is the temperature, $A$ is the pre-exponential coefficient, $E_a$is the activation energy of reaction , $R$ is the universal gas constant, $n$ is the concentration exponents and the quantity in bracket is the concentration. The fuel concentration $[fuel]$ contains CO, $H_2$and OH (i.e. $[fuel]$ = $[CO + H_2 + OH]$). The final term in the equation $[gas]^{n_{gas}}$ is the gas concentration which effects the third body reactions to be taken in the reaction rate model[19].

### 2.3.3 Global reaction rate parameter fits

The parameters of global reaction $A$, $E_a$, $n_{fuel}$, $n_{O_2}$ , and $n_{gas}$ are permanent by using the reaction kinetics mechanism applied by using the python package Cantera[19].In particular, the 13 species and 28 reaction mechanism are used in [41] , which was proposed to the model of combustion chamber in solid rocket motor plumes to adjust the parameters of the model.

## 2.4 Influence of Fuel Composition and Engine Design

Rocket motors are integral to space exploration, military applications, and scientific research, driving technological advancements in propulsion systems. The performance, efficiency, and reliability of rocket motors are significantly influenced by two key factors: fuel composition and engine design. These factors play a pivotal role in determining thrust, specific impulse, combustion efficiency, and overall mission success.

Fuel composition dictates the energy content, combustion characteristics, and environmental impact of a rocket motor. Liquid propellants, such as oxygen and liquid hydrogen offer high efficiency and clean combustion but need difficult storage and handling systems. Solid propellants frequently composed of a oxidizer, fuel and binder, arrange for easiness and consistency but lack the controllability of their liquid counterparts. Hybrid fuels is combined elements of both liquid and solid systems are emerging as a promising compromise, contribution protection and flexibility benefits. On the other hand, engine design affects how successfully the energy from the propellant is changed into thrust. Elements such as chamber pressure, nozzle geometry, cooling mechanisms, and material choice affect engine efficiency and stability. For example, improvements in additive manufacturing have supported the production of intricate cooling channels and optimized nozzle designs, improving thermal resistance and thrust efficiency [42].

## 2. Fluid Dynamics of Exhaust Plumes

The examination of the Fluid Dynamics of Exhaust Plumes encompasses the analysis of the behavior, flow characteristics, and interactions of gases released from a source, including but not limited to jet engine, rocket, or industrial chimney, with their ambient environment. This field is critical in understanding how exhaust gases perform in terms of velocity, temperature, pressure, turbulence, and chemical composition as they transfer through the atmosphere or other channels.

Exhaust plumes contain multiple flow regimes, including compressible flows, turbulence, and chemical reactions, depending on the propulsion system and operational environment. For instance, in rocket propulsion, under-expanded plumes act together with ambient pressure to form features like shock waves and expansion fans, significantly impacting the efficiency and thermal belongings on surrounding structures. To examine the plume flow fields in the vacuum by using some methods, the Direct Simulation Monte Carlo (DSMC) method has been frequently used. By solving the Boltzmann equation using statistical methods, the flow of complex transition systems and free molecules in the far field at the outlet of the propellant nozzle can be exactly simulated, as shown in Figure 5. [43] [44].

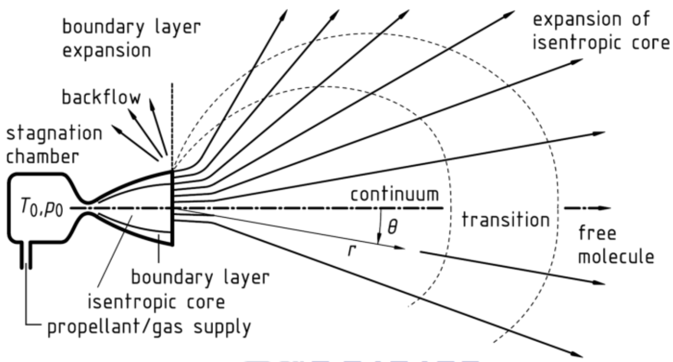


**Fig.5 Typical modes of the flow of loops in vacuum [44].**

Advances in computational fluid dynamics (CFD) have allowed complete simulations of plume behaviors, accounting for complex gas dynamics, reactive flows, and multi-phase interactions. These tools help expect plume expansion, thermal loads, and pollution risks, mostly in space applications where such factors influence spacecraft performance [44] [45].

### 2.1 Vortex Dynamics and Shear Layers

The study of vortex dynamics and shear layers in rocket motors is essential for understanding the complex fluid-structure interactions and instabilities that arise during rocket propulsion. These phenomena significantly influence the efficiency, stability, and acoustic behavior of rocket engines.

**Vortex dynamics** involve the formation and evolution of vortices, which are rotational flows of fluid. In rocket motors, vortices are generated due to sudden changes in geometry, such as nozzle throats or combustion chamber interfaces. These vortices can interact with acoustic waves and lead to pressure oscillations, impacting engine performance. Studies have shown that vortex shedding frequencies in such systems are influenced by factors like flow velocity, chamber geometry, and acoustic feedback mechanisms [46] [47]. Combustion chamber for SRM consist of two or more fuel blocks combined by inhibitors where fuel burn slowly and form discontinue structures at junction of fuel segments by producing vortex shedding [4] . Vortices generated in chambers due to hydrodynamic instabilities are shown in Fig. 6.

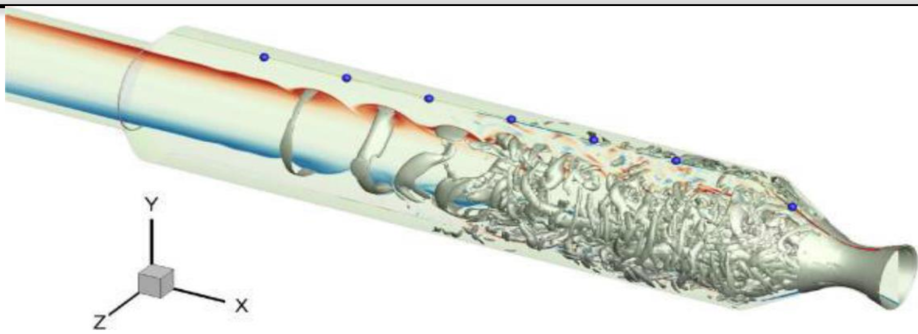


**Fig. 6 Vortex generation in combustion chamber.**

In Solid Rocket Motors three kinds of vortex shedding have been observed such as obstacle, partial/surface and corner/angle vortex shedding.

i. Obstacle Vortex Shedding (OVS): generated in combustion chamber are actually because of protruding obstacles placed inside the chamber such as inhibitors [48].

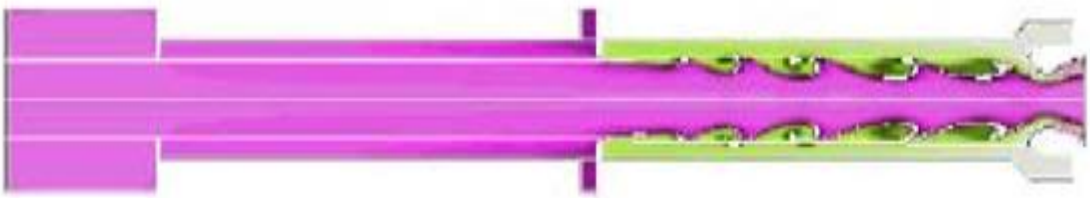

**Fig. 7 Obstacle Vortex Shedding [48]**

ii. Angle (Corner) Vortex Shedding (AVS): in combustion chamber are because of irregularities of geometry caused angle/corner vortex shedding such as grooves in grain and the backward-facing step [48].

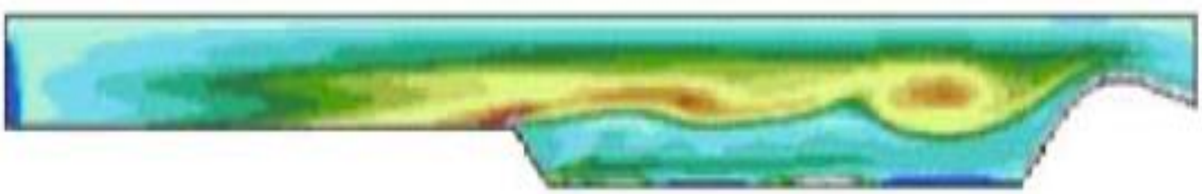

**Fig.8 Angle Vortex Shedding [48].**

iii. Surface/Parietal Vortex Shedding (SVS): due to obstacles or cavities generates slightly pressure oscillations where the main source of disturbance exists in SVS which move through wall is injection of fluid for combustion [48].

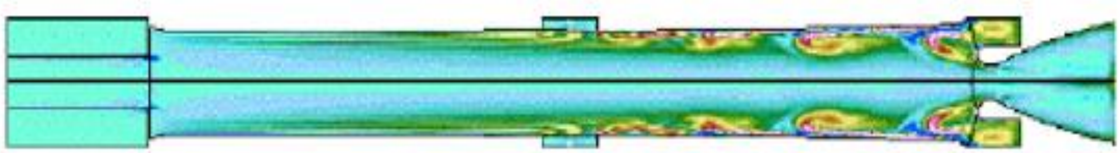

**Fig.9 Surface Vortex Shedding [48].**

Shear layers, which form at the interface between two fluid streams moving at different velocities, are crucial in defining the stability and turbulence within the combustion chamber. Instabilities in shear layers can amplify acoustic waves, leading to phenomena like screech tones or low-frequency oscillations. These effects are particularly prominent in hybrid and solid rocket motors, where interactions between shear layers and combustion dynamics are complex.

### 3.2. Turbulence and Instability Mechanisms

Turbulence and instability in rocket motors are critical phenomena that significantly influence their performance, safety, and reliability. In rocket combustion chambers, high-pressure and high-temperature conditions, coupled with rapid combustion reactions, generate complex fluid dynamics characterized by turbulent flows and instabilities. These behaviors can amplify acoustic oscillations, lead to structural damage, and compromise thrust efficiency.

Turbulence in rocket motors typically arises from the interactions between high-speed propellant streams and the chamber walls or other structures, creating chaotic, energy-dissipating vortices. These vortices play a role in mixing fuel and oxidizer, enhancing combustion efficiency. However, when unsteady interactions between combustion and acoustic waves occur, instabilities can arise. These instabilities, particularly combustion-driven acoustic instabilities, manifest as oscillatory pressure fluctuations, which can potentially grow in amplitude and disrupt stable operation [49] [50]. Key instability mechanisms include interactions between the combustion process and resonant acoustic modes of the chamber. This coupling can result in pressure waves that amplify through positive feedback, a phenomenon seen in transverse and longitudinal instabilities. External factors, such as injector design, chamber geometry, and nozzle configuration, significantly impact these instabilities, necessitating advanced computational modeling and experimental analysis to mitigate risks. Understanding and controlling these mechanisms are essential for improving rocket motor design. Techniques like passive damping (using resonators) and active control (feedback systems) have been employed to suppress instabilities, ensuring stable and efficient rocket operation [51] [52] [53].

### 3.3. Mach Number Influence on Plume Structures

The behavior and structure of exhaust plumes produced by rocket motors are crucial to understanding and optimizing the performance and environmental impact of these propulsion systems. Rocket exhaust plumes are difficult fluid dynamic phenomena considered by high-temperature, high-velocity gas flows interacting with surrounding atmospheric situations. One of the critical parameters inducing the plume structure is the Mach number, which measures the ratio of the exhaust gas velocity to the speed of sound in the local medium.

The Mach number significantly affects the shock wave designs, flow increase, and mixing of exhaust gases with the air. At subsonic Mach numbers, the plume tends to remain relatively involved and symmetric. In contrast, at supersonic and hypersonic regimes, the plume exhibits different structures such as expansion fans, shock diamonds, and shear layer uncertainties, which contribute to increased turbulence and mixing. The evolution from one regime to another is accompanied by noticeable changes in plume morphology, affecting factors like thrust efficiency, thermal signatures and chemical diffusion in the surroundings. Moreover, the Mach number's influence on plume dynamics plays a important role in applications such as spacecraft design, launch safety and eco-friendly assessments. For instance, understanding how Mach number differences affect plume dispersion helps mitigate acoustic and thermal loads on launch platforms and nearby structures. The missile-based thermal atmosphere has a configuration with twin-nozzle with free flow of Mach numbers from 0.6 to 3.0. The results show that the number of free flow machines has a significant impact on the thermal environment on a missile-based basis at the transonic stage. The increase of Mach number from 0.6 to 1.0 reasons the convective heating of the rocket base to increase by 7.7 times. This situation occurs due to the plume-induced shock waves generated by rocket motor caused by the ultrasonic free layer of change and the motor gradient while the flight speed exceeds the speed of sound [54].

### 3.4. Computational Modeling and Simulation Techniques

In computational fluid dynamics modeling and simulations are key tools to understand the flow behavior of a rocket motor and they refer to how exhaust gases move and interact as they pass through the nozzle of motor and into the air. In Computational Fluid Dynamic Computer modeling and simulation techniques are the key tools to understand the intricacies of afterburning in rocket motor. These techniques help the researcher to understand the relations between fluid dynamics, heat transfer and chemical kinetics that affect afterburning in the rocket motor. Chemical reaction mechanisms are working to simulate the evolution of exhaust plume with many conditions by using computational fluid dynamic techniques. Advanced turbulence models, such as large eddy simulations (LES) and Reynolds-averaged Navier-Stokes (RANS) methods are used to employ the turbulent mixing of exhaust gases with surrounding environments, which is a crucial parameter in after burning. Furthermore, simulation of turbulence flow, intricate chemical kinetics. It shows the main role in the combustion chamber that happens during the time of afterburning. These chemical reactions contain a wide range of species and reactions, which require the use of simplified or reduced chemical models for computational efficiency without reducing accuracy. The modeling of radiative heat transfer is also integrated into simulations to incorporate the thermal emissions emanating from the high-temperature plume, which is difficult for evaluating environmental impacts and thermal regulation in proximity to launch sites [55].

## 4. Environmental Impacts of Afterburning Exhaust

Afterburning exhaust plume can have many environmental impacts such as air pollution, noise pollution, thermal pollution etc. The increasing fuel consumption and emissions are associated with afterburners, which is commonly used in military jets and high-speed aircraft.

The environmental impacts of rocket launches are an area in which space exploration and satellite deployments continue to rise. The afterburning of exhaust plumes of rocket motor play a significant eco-friendly challenge among the many emissions. Afterburning is the secondary combustion of unburned fuel and exhaust products. Unburned fuel and exhaust produce harmful gases in the environment. This process modifies the chemical composition of the plume but also improves its thermal and radiative characteristic, they impact the surrounding environment in various manners. The main environmental significance of afterburning is the emission of greenhouse gases (GHGs), including carbon dioxide($CO_2$) , water vapor($H_2O$) and other suggestion gases, such as nitrogen oxides ($NO_x$) and chlorine compounds, which are dependent on the propellant type. These greenhouse gases contribute to environmental pollution and can exacerbate global warming. Furthermore, high energy emissions in the upper atmosphere can reduce the ozone layer, as they observe with chorine which contain the exhaust products from rocket motors [56].

The thermal effect of after burning is the high-temperature exhaust gases which can disrupt atmospheric stratification and potentially effect weather designs in the area of common launch places. The properties of plume's radiative contribute the generation of infrared and ultraviolet radiation, which may have suggestions for both eco-friendly and observational studies of the air [57]. Furthermore, the atmospheric effects of afterburning exhaust at or near launch places of rocket is the localized environmental influences such as soil and water pollution. The deposition of unburned propellants, heavy metals and acidic residues can damage ecosystems and pose risks to human health. These influences highlight the afterburning processes to grow mitigation strategies and certify maintainable aerospace processes.

### 4.1. Emission of Greenhouse Gases and Particulates

The emission of greenhouse gases and particulates of rocket motor is a significant environmental concern as it contributes to global warming, air pollution and human health issues. Rocket propulsion systems play an important role in modern space exploration and the deployment of satellites; however, their emissions of gases of rocket have generated important environmental concerns. Rocket exhaust plumes are difficult mixtures of chemical species, including greenhouse gases (GHGs) such as carbon dioxide($CO_2$), Nitrous Oxide ($N_2O$) and water vapor($H_2O$), as well as particulate matter like soot and metal oxides. These emissions have the possibility to impact climate, atmospheric chemistry and local ecosystems, mostly as the frequency of takeoffs increases. Greenhouse gases produced during rocket takeoffs contribute to global warming by improving the Earth's radiative forcing. While carbon dioxide ($CO_2$) emissions from rockets are comparatively small compared to other sectors like flight and energy production, water vapor ($H_2O$) inserted directly into the stratosphere by rocket engines can have a extremely large warming effect. Unlike lower-atmospheric water vapor, which has a short residence time, stratospheric water vapor continues for extended periods and plays a serious part in altering radiative equilibrium and ozone chemistry. Particulates in rocket exhaust, such as soot and aluminum oxide, are another environmental concern. Soot particles contribute to climate warming by absorbing solar radiation, while aluminum oxide from solid rocket motors can act as nucleation sites for ice cloud formation, potentially affecting local and global weather patterns. Furthermore, these particles affecting environments and human health at the launch places of rocket. [58] [59]. The creation of soot and emissions of gases of rocket motor in the exhaust plumes was known as a significant environmental problem. The arrangement and spreading of these particles contribute to global warming and air quality degradation [60].

**4.2. Chemical Reactions in the Atmosphere**

The emissions of rocket propulsion are complex due to chemical reactions. It is clarified that the chemical processes are involved and it is imperative to begin within the combustion chamber of rocket motor, also analysis the subsequent nozzle expansion phenomena. Upon the growth of the gases through the nozzle, interaction and reaction of gases have happened with in environment. These reactions not only change the chemical composition of the exhaust plume of rocket motors but also have implications for the surrounding environment, including the stratosphere and troposphere. The specific chemical reactions which occur during after burning of exhaust plume is necessary for evaluating the environmental impacts of rocket launches, such as ozone depletion, emissions of greenhouse gases and chemical formation. The use of numerical simulations is required to explain the process and chemistry from the combustion chamber of rocket motors. The rocket plume is divided into two parts: one is hot plume and 2nd is cold plume. In the study of hot plumes, the chemistry and fluid dynamics of rocket plumes are modeled from the combustion chamber through the rocket nozzle and from the point where the temperature and pressure of the plume equals to the atmosphere. Chemical equilibrium is supposed in the combustion chamber. But, as the flow expands into the nozzle, one-dimensional finite rate chemistry is accepted. In the cold plume system, high-temperature homogeneous gas phase reactions become irrelevant, and photo chemically introduced reactions are of main concern. In addition, species movement is dominated by the diffusion process in the cold plume regime [61] [39] .

**4.4.1 Four key atmospheric chemical reactions dominate the afterburning process:**

i. Oxidation of Carbon-Containing Compounds: Unburned hydrocarbons and carbon monoxide ($CO$) in the exhaust plume react with oxygen ($O_2$) and hydroxyl radicals ($OH$) to form carbon dioxide ($CO_2$) and water($H_2O$) . For example, ($CO$) reacts as follows:

$$CO + OH \rightarrow CO_2 + H \quad (10)$$

This reaction is crucial for converting incomplete combustion products into stable compounds, although it also contributes to the greenhouse gas inventory.

ii. Water Vapor Formation: Hydrogen-containing species such as $H_2$ and H react with atmospheric oxygen to produce water vapor:

$$2H + O_2 \rightarrow 2OH \quad (11)$$

$$2H_2 + O_2 \rightarrow 2H_2O \quad (12)$$

Water vapor is a significant byproduct of rocket afterburning, particularly for hydrogen-based fuels, and its injection into the stratosphere can influence radiative forcing and ozone chemistry [62].

iii. Nitrogen Oxide ($NO_x$) Formation: High-temperature conditions in the exhaust plume promote the reaction of atmospheric nitrogen ($N_2$) with oxygen, forming nitrogen oxides:

$$N_2 + O_2 \rightarrow 2NO \quad (13)$$

$$NO + O \rightarrow NO_2 \quad (14)$$

$NO_x$ species play a major role in ozone depletion and the formation of acid rain. Their persistence in the atmosphere makes them a critical component of rocket exhaust emissions.

iv. Ozone Depletion Reactions: Chlorine-containing compounds in the exhaust plume, especially from solid rocket propellants, catalytically destroy ozone in the stratosphere. For instance, reactions involving chlorine radicals (CI) proceed as follows:

$$CI + O_3 \rightarrow CIO + O_2 \quad (15)$$

$$CIO + O \rightarrow CI + O_2 \quad (16)$$

These reactions form a catalytic cycle that can deplete significant amounts of ozone, impacting the ozone layer's ability to shield the Earth from ultraviolet radiation [63].

### 4.3. Mitigation Strategies for Emission Reduction

As space exploration and commercial satellite launches continue to expand, the environmental impact of rocket propulsion systems is gaining increased attention. Rocket exhaust emissions, including greenhouse gases (GHGs), particulates, and ozone-depleting substances, present significant challenges to both local ecosystems and global atmospheric chemistry. These emissions can contribute to reducing air quality, interrupting the ozone layer and climate change, requiring the growth of effective mitigation approaches to minimize their influence while supporting the development of the aerospace sector. Mitigation approaches for emission decrease in rocket propulsion focus on three main approaches: developing cleaner propellants, optimizing propulsion system design, and implementing operational improvements.

#### 4.3.1 Cleaner Propellants:

Transitioning to natural propellants is a vital approach for decreasing dangerous emissions. For example, liquid hydrogen $(LH_2)$ matching with liquid oxygen $(LO_2)$ produces mostly water vapor as a result, creating a cleaner alternative to kerosene-based fuels or solid propellants that emit soot, carbon dioxide $(CO_2)$ and chlorine-containing combinations. Furthermore investigation interested in green propellants such as ammonium dinitramide (ADN)-based formulations goals to decrease toxic residues and GHG productions [64].

#### 4.3.2 Propulsion System Optimization:

Combustion efficiency and engine performance can decrease incomplete combustion and minor emissions of rocket nozzle design. Inventions such as aero spike nozzles, staged combustion cycles, and regenerative cooling increase fuel efficiency and reduction the creation of pollutants like soot and nitrogen oxides $(NO_x)$.

#### 4.3.3 Operational Improvements:

The enhancement of launch trajectories, reducing unnecessary payload mass, and applying stricter rules on takeoff frequency can together limit the eco-friendly footprint of rocket takeoffs. For example, minimizing launch frequency in sensitive areas or seasons could mitigate local atmospheric and ecosystem disturbances.

## 5. Advances in Experimental and Computational Methods

Experimental and computational techniques have significantly enhanced the ability to describe and predict rocket exhaust plume performance. Experimental methods, such as high-speed imaging, spectroscopic diagnostics and particle analysis allow for full measurement of plume properties in simultaneous and under different operating situations. These techniques deliver critical data on temperature, pressure, chemical structure and particulate circulation, allowing deeper vision into the physical and chemical processes governing plume dynamics [65] [66] [67]. On the computational and numerical simulations leveraging high-fidelity models and advanced computational fluid dynamics (CFD) tools have become necessary. These techniques simulate difficult multi-phase and reactive flow fields, incorporating detailed chemical kinetics and radiative heat transfer. Coupling CFD with experimental data simplifies validation and modification of analytical models, improving their consistency in real-world uses. This combination of experimental and computational methods addresses the tasks modeled by the extreme situations contained by rocket plumes, together with high temperatures, supersonic velocities, and turbulent relations. The continuing combination of developing tools, such as machine learning algorithms and high-performance computing, promises to revolutionize the field by allowing rapid, large-scale simulations and real-time diagnostics.

### 5.1. Laboratory-Scale Testing of Exhaust Plumes

The plume of a rocket motor is a dynamic and difficult system, containing high-temperature gases, particulate matter and radiative emissions produced during thrust. Considering the performance of exhaust plumes is necessary for improving rocket performance, mitigating environmental effects and make sure safety in takeoff processes. Although complete testing is the furthermost direct method to examine the rocket exhaust plumes, it is frequently unreasonable due to high prices, security concerns and logistical tests. As a outcome, laboratory-scale analysis has emerged as a energetic tool for plume classification, offering controlled environments to study plume performance and validate computational simulations.

Laboratory-scale analysis contains simulating the combustion and flow dynamics of rocket motors under scaled-down situations. By repeating vital features such as chamber pressure, nozzle design and fuel composition, these tests provide insights into the physics of plume creation and spread. Advanced diagnostic methods, containing laser-induced fluorescence (LIF), particle image velocimetry (PIV) and infrared spectroscopy, enable precise measurement of plume properties, such as velocity fields, temperature and chemical species attentions [1] [68].

### 5.2 High-Fidelity CFD Approaches

To improve the accuracy of CFD simulations by using High-fidelity CFD (computational fluid dynamics) techniques. They can reproduce difficult flow dynamics and relations, such as vortices, shocks and separation. This can lead to more accurate forecasts of masses like drag, lift and heat transmission.

High-fidelity CFD techniques contain to solving the full Navier-Stokes equations with advanced turbulence simulations, chemical reaction mechanisms and radiative heat transfer effects. Distinct simplified methods, such as empirical models or reduced-order models, high-fidelity CFD captures complex parts of multi-phase flow, shock structures and mixing methods in rocket motors. This level of part is vital for understanding phenomena like combustion instabilities, nozzle erosion and thermal defense system performance [69]. The maximizing availability of high-performance computing resources has significantly expanded the scope of high-fidelity CFD in rocket motor investigation. Methods such as large-eddy simulation (LES) and direct numerical simulation (DNS) allow researchers to resolve fine-scale turbulence and temporary phenomena with significant accuracy [69] [70]. Furthermore, multi-scale modeling frameworks combine high-fidelity CFD with lower-fidelity methods to efficiently simulate full-scale systems while retaining critical particulars in local areas.

### 5.3 Advances in Diagnostic Tools (e.g., Laser-Based Techniques)

Rocket engine improvement and maintenance needs accurate diagnostic tools to confirm optimal performance, safety and consistency. Current progresses in diagnostic methods, mostly laser-based methods, have transformed the field by offering non-intrusive, highly accurate methods for exploring combustion, physical integrity and structural performance in real-time. These improvements report critical challenges in understanding complex phenomena such as combustion instabilities, thermal stress and erosion in rocket thrust systems.

Laser-based diagnostic tools like Laser-Induced Breakdown Spectroscopy (LIBS), Coherent Anti-Stokes Raman Spectroscopy (CARS) and Particle Image Velocimetry (PIV) have begun as powerful methods for describing rocket motor behavior. LIBS, for example, allows for the in-situ analysis of physical structure, aiding in monitoring erosion and wear in solid rocket motors. CARS is mostly effective in computing temperature and species concentration in high-temperature combustion surroundings, providing important data for improving fuel performance. Also, PIV proposals detailed insights into flow dynamics, critical for considerate turbulent interactions within rocket nozzles and combustion chambers [71] [72].

### 5.4. Integration of LES and URANS in Plume Studies

Rocket motor plume studies are critical for understanding exhaust dynamics, thermal effects and environmental connections, mostly for improving propulsion systems and making sure safety in aerospace uses. Usually, computational fluid dynamics (CFD) techniques like Reynolds-Averaged Navier-Stokes (RANS) have been broadly used for plume modeling due to their efficiency in providing time-averaged solutions. However, RANS models are often limited in capturing temporary, small-scale turbulent structures that significantly impact the plume's performance. To address these limitations, the combination of Large Eddy Simulation (LES) and Unsteady Reynolds-Averaged Navier-Stokes (URANS) techniques has emerged as a promising method for improving the fidelity of plume simulations.

## 6. Mathematical Formulation

To simulate the afterburning effect on the rocket exhaust plume, the supersonic flow model is established based on the RANS equations, and then the finite-rate chemical kinetics is used to calculate the secondary combustion of carbon monoxide and hydrogen. The three-dimensional governing equations for mass, momentum, and energy are presented by the following[73]

$$\frac{\partial U}{\partial t}+\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}=\frac{\partial G_1}{\partial x}+\frac{\partial G_2}{\partial y}+\frac{\partial G_3}{\partial z} \qquad 1(7)$$

Where $U$ is conservative variable, $F_1, F_2, F_3$ are the inviscid flux terms and $G_1, G_2, G_3$ are the viscid flux terms.

These variables can be written as:

$$\begin{cases} U=(\rho,\rho u,\rho v,\rho w,\rho e,\rho Y_i)^T \\ F_1=\left(\rho u,\rho u^2+p,\rho uv,\rho uw,\rho u\left(e+{}^{p}/_{\rho}\right),\rho uY_i\right)^T \\ F_2=\left(\rho v,\rho uv,\rho v^2+p,\rho vw,\rho v\left(e+{}^{p}/_{\rho}\right),\rho vY_i\right)^T \\ F_3=\left(\rho w,\rho uw,\rho vw,\rho w^2+p,\rho w\left(e+{}^{p}/_{\rho}\right),\rho wY_i\right)^T \end{cases} \qquad (18)$$

$$\begin{cases} G_1=\left(0,\tau_{xx},\tau_{xy},u\tau_{xx}+v\tau_{xy}+w\tau_{xz}+\frac{K\partial T}{\partial x},\rho D\frac{\partial Y_i}{\partial x}\right)^T \\ G_2=\left(0,\tau_{yx},\tau_{yy},u\tau_{xy}+v\tau_{yy}+w\tau_{yz}+\frac{K\partial T}{\partial y},\rho D\frac{\partial Y_i}{\partial y}\right)^T \\ G_3=\left(0,\tau_{zx},\tau_{zy},u\tau_{xz}+v\tau_{yz}+w\tau_{zz}+\frac{K\partial T}{\partial z},\rho D\frac{\partial Y_i}{\partial z}\right)^T \end{cases} \qquad (19)$$

Where $u, v, w$ are the velocity components of $x, y$ and $z$ directions, $Y_i$ is the mass friction of species and D is the diffusivity constant.

When the velocity of four-engine rocket exhaust plume was speeded up to 5Mach number, the significant non-linear inertial effect in the Navier–Stokes equations tend to overcome the damping effects of viscosity. The material effect that these small-scale turbulent motions have on the mean flow is usually modeled with a separate stress field. In such situations, it is common to apply the concepts of either time averaging (an integral average over a sufficiently long period of time) or ensemble averaging (an average at the same instant, over a repeated number of realizations). The stress fields so obtained allow a description of the averaged effects of momentum, thermal and species mixing due to turbulence. In the realizable $k-\epsilon$ turbulence model, the Boussinesq relation is used to obtain Reynolds-stresses from the modeled eddy viscosity $\mu t$ and the available mean-strain tensor:

$$\tau_{ij}=\rho\overline{\mu_i\mu_j}=\frac{2}{3}\delta_{ij}\rho k-\mu_t S_{ij} \qquad (20)$$

Where

$$S_{ij}=\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_{x_X}}-\frac{2}{3}\frac{\partial U_k}{\partial x_k}\delta_{ij} \qquad (21)$$

The model consists of the following transport equations for the kinetic energy k and the turbulent dissipation rate $\epsilon$ [74].

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_j}(U_j\rho k)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\rho\varepsilon \quad (22)$$

And

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_j}(U_j\rho\varepsilon)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial\varepsilon}{\partial x_j}\right]+\frac{1}{T_t}(C_{\varepsilon1}P_k-C_{\varepsilon2}\rho\varepsilon) \qquad (23)$$

The constants of this model are given by:

$$C_{\varepsilon1}=1.44,\ C_{\varepsilon2}=1.92,\quad \sigma_k=1.0,\ \sigma_\varepsilon=1.3 \qquad (24)$$

## 6. Challenges and Future Directions:

The study of rocket exhaust plume of rocket motor is a difficult because it contains fluid dynamics, thermodynamics, chemical kinetics and radiative transfer. Despite the significant improvements, some problems continue in the understanding and mitigating of the effects of rocket exhaust plumes. Important parts of concern contain plume-induced thermal and mechanical loads on rocket components, eco-friendly effects such as the deposition of hazardous materials in the air and acoustic and optical interference during takeoff. Additionally, the increased use of advanced propulsion systems with hybrid and green propellants introduces additional uncertainties that require full verification and modeling.

The future of exhaust plume of rocket will rely on the improvement of new computational tools, such as high-fidelity simulations and advanced diagnostic techniques. These methods' goal to expand the accuracy of plume characterization under different operating conditions, minimize environmental influences and improve the safety of aerospace tasks. Furthermore, the increasing focus on reusable launch systems and in-situ resource utilization for space exploration increases the need for in-depth studies on exhaust plume connections in both terrestrial and extraterrestrial environments. Addressing these challenges is critical for advancing the aerospace industry's sustainability and ensuring the reliability of next-generation space missions.

### 6.1. Multi-Scale and Multi-Physics Modeling of Plumes

The combination of methods from different scientific disciplines to study the complex interactions between physical processes and phenomena. Multi-scale and multi-physics modeling is important in several engineering applications, such as in the design of power plant of nuclear. Multi-physics and Multi-scale modeling contain relationship between mechanical, thermal, electrical, magnetic, chemical and moisture-absorption induced loads/effects on materials and configurations using various length and time scales. Multi-physics field allows the simultaneous and modeling of multiple physical phenomena in distinct area in a similar simulation structure. This system allows for a complete understanding of complex systems in which various physics work together for example fluid dynamics, chemical reactions and heat transfer. Relations of Different physical phenomena are taken, multi-physics leads to more accurate simulations that better reflect real conditions. Furthermore, multi-physics promote the complex design of difficult systems by seeing the joint effects of unalike physics phenomena on the structure performance. However, multi-objectives are computational strengths and require important computing resources to accurately solve related equations. These physical phenomena can lead to numerical instabilities. System control of this instability requires detailed selection of numerical methods, discretization structures, sampling parameter structures and solver settings to confirm strong and exact solutions. In addition, multi-physics can be more difficult to configure and verify due to the improving complexity of the simulations and their relationships. The Integrating multiple physical cases requires the possibility for many disciplines to confirm the accuracy and consistency of simulations [75].

### 6.2. Addressing Nonlinear Couplings in Combustion and Aerodynamics

In high-speed flow systems, the interaction between the combustion processes and aerodynamics behavior is complex. This interaction leads to nonlinear coupling, which significantly affects system stability, performance and efficiency of propulsion systems.

Nonlinear phenomena, such as acoustic instabilities, vortex shedding and combustion dynamics, can increase due to response mechanisms between aerodynamic forces and non-steady combustion. These connections can lead to unbalanced performance, with importance, loss of reliability and physical damage in extreme cases.

Latest results in the field of computational fluid dynamics (CFD) and experimental techniques have allowed scientists to examine these nonlinear combinations with more accuracy. Numerical techniques such as large eddy simulations (LES) and direct numerical simulations (DNS) have proved effective in taking the fine-scale parts of turbulence and combustion relations in rocket engines. Experimentally, high-speed imaging and advanced spectroscopy have providing valuable insights into the transient performance of reacting flows and aerodynamic fields [76]. These advances challenges are continued. The interaction between thermal expansion, a change in pressure and the instability of the flow often introduces high -line effects, which are difficult to predict and mitigate. For example, as well known in research, the relationship between transverse acoustic mods and non-steady heat release in solid rocket engines can increase instability[26] and [77] . Addressing these tasks needs a multidisciplinary methodology that integrates insights from fluid mechanics, thermodynamics and physical dynamics.

### 6.3. Development of Eco-Friendly Propellants and Fuels

The increasing eco-friendly concerns associated with conventional fuels and propellants have determined the scientific community to explore maintainable alternatives that decrease ecological influence while maintaining high performance. Traditional fuels, such as hydrocarbons and hydrazine-based compounds, are effective but frequently release harmful chemicals like carbon dioxide ($CO_2$), nitrogen oxides $NO_x$ and particulate substance during burning [22] . Furthermore, the toxicity and hazardous handling requirements of conventional propellants position significant risks to both human health and the environment [78]. Eco-friendly propellants and fuels are developing as favorable solutions to these challenges. These replacements leverage renewable feed stocks, green chemistry principles and innovative technologies to decrease emissions and reduce dependency on fossil assets [79]. The growth of such maintainable energy sources is not without its challenges. Balancing energy thickness, cost-effectiveness and scalability remains a main focus of continuing research. Additionally, the integration of eco-friendly fuel types into existing infrastructure requires for extensive testing and modifications [80]. Even with these obstacles to improving the renewable energy materials science and technology sector, they continue to cover the path of advanced solutions.

### 6.4. Role of Machine Learning in Plume Prediction and Control

Ideal prediction and control of plumes produced through thrust and combustion processes, such as aerospace engineering, industrial emissions management and environmental observations, is important for various applications. Plumes of rocker motors are composed of gases, particles and other by-products, they can significantly impact operational efficiency, safety and environmental efficiency. Old methods for modeling and monitoring exhaust plume performance frequently depend on computational fluid dynamics (CFD) simulations and experimental techniques, they are computationally intensive and time-consuming [81]. Machine learning (ML) is a branch of artificial intelligence, which offers a transformative technique to report these challenges. By Using large datasets and advanced algorithms, machine learning (ML) methods can model nonlinear complex systems with remarkable speed and accuracy. The machine learning (ML) model's plume prediction field analyzed the dynamics of combustion, determined the characteristics of the exhaust plume and confirmed its ability to predict dispersion patterns in various situations.
[82].

## 7. Conclusion

The study of rocket exhaust plume dynamics has evolved significantly with advancements in computational and diagnostic techniques. High-fidelity CFD methods, such as LES and URANS, provide more accurate simulations of plume behavior, improving predictions of turbulence, heat transfer, and chemical interactions. The integration of these models with advanced diagnostic tools, particularly laser-based techniques, has enhanced the validation of computational results and refined propulsion system designs. However, challenges such as the accurate representation of extreme thermodynamic conditions, plume-induced structural effects, and environmental concerns remain critical areas for further investigation. Addressing these issues through improved simulation frameworks, experimental validation, and sustainable propulsion strategies will be essential for advancing next-generation aerospace propulsion systems. Future research should also emphasize the role of reusable launch systems and alternative propellant technologies to minimize the environmental footprint of space missions.

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