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EXPERIMENTAL AND NUMERICAL STUDY ON THE RISE OF BUBBLES IN THE VERTICAL WATER CHANNELS

Sannan Aziz[*1], Kareem Akhtar[2], Nadeem Rehman[3], Mohsin Ali[4]

[*1,2,3,4] *Department of Mechanical Engineering, University of Engineering and Technology, Peshawar, Pakistan*

[*1]sannanaziz.mec@uetpeshawar.edu.pk

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Corresponding Author: *
Sannan Aziz

Abstract
This work investigates numerically and experimentally the behavior of air bubbles rise i.e. path and motion in vertical water channels using a submerged needle and the impact of water channel size on the behavior of rising air bubbles and contrasts this current threedimensional (3D) study with the earlier 2D studies. Experimental investigations were taken by installing two high-speed digital cameras (HSC's). For smaller flow rates, the bubble's trajectories have been observed to spiral rather than rectilinear. The volume of fluid (VOF) approach is used for numerical analysis to investigate the trajectory of air bubbles in different water channels. The bubble trajectories of the front (x-y) plane and superimposed view of the (y-z) plane recorded during experimental investigations are drawn using MATLAB. Then graphical representations are drawn in MS EXCEL. The bubbles rise and follow a spiral path. Water channel size does not affect the behavior of rising air bubbles.

INTRODUCTION

The simultaneous movement of two or more phases (such as liquid, gas, or solid) within a system is referred to as multiphase flow. It is a dynamic and intricate phenomena that arises in a number of industrial and natural processes. In disciplines including geophysics, environmental science, petroleum engineering, and chemical engineering, an understanding of multiphase flow is essential. Numerous numerical, experimental, and theoretical investigations have examined the movement of bubbles via pipes, tubes, and channels. Because of their many applications, which range from micro to macroscale flows, for example Al-Otaibi et al. [1] performed particle-in-cell multiphase simulations for reactors running in different reactive flow regimes. According to the data, the reactor performed best in the turbulent fluidization regime in terms of CO/H2 ratio, CO+H2 productivity, and CH4 and CO2 output. Conversely, the response performance was lowest in the bubbling regime. This study highlighted how important flow hydrodynamics is in determining the performance of fluidized-bed reactors work in the dry reforming of methane. Thermosiphons are used for cooling of chemical reactors, biological applications, microfluidics, mixing in microfluidics, and other uses [2] [3] [4]. During different phases of manufacturing, air-water two-phase flow may be used in chemical reactors and processing units. It is essential to understand the flow characteristics to maximize product yields and reaction kinetics [5]. Controlling the air-water two-phase flow thus requires a thorough understanding of the bubbles' sizes and trajectories. The rise of the air bubbles is strongly dependent on their sizes as well as gas-liquid characteristics like density, surface tension, and viscosity [6]. Aeration system clogging, a common issue with many diffuser types, is prevented by the even distribution of air bubbles and the generation of tiny ones, which increases the oxygen mass transfer surface [7]. Thus, tank shape, dispersion coefficient, and gas velocity all affect the distribution of dissolved oxygen content.

A thorough understanding of the creation and movement of air bubbles in the water is essential for optimizing the biological treatment process. To improve active sludge processes through aeration, research has started by examining the oxygen transfer process. It has been discovered that when bubble size decreases oxygen transfer efficiency rises [8]. X.F. Liu et al. [9] conducted an experimental study on the characteristics of air-water two-phase flow in a vertical helical rectangular channel. It was established that the properties of the flow in a helical rectangular channel are primarily influenced by centrifugal force, inertia force, pressure gradient, and buoyancy force. Andrea Cioncolini, Micro Magnini [10] investigated that the bubble's qualitative behavior, which showed a wavering rise dynamic similar to that of bubbles rising through unconfined liquids, was unaffected by the annular channel's confinement. When compared to bubbles rising through unconfined liquids, the shape and rise velocity of the confined bubbles were affected; they climbed more slowly and with less deformation. The air accumulating in the hole, which takes the form of a funnel, is caused by the air injection via the hole at the bottom of the water tank. The top of the funnel-shaped area is where the air bubbles are separated and progressively gather. The vortex structure is connected to the way air bubbles separate. The backflow created by the vortices rotating in the opposite direction on each side of the center lines encourages the separation of bubbles. The streamlines move in a zigzag pattern due to variations in vortex intensity [11].

The motion of the bubble, the behavior of the interface, and the local variables of the air-water two-phase flow system in a three-dimensional reducer pipeline filled with liquid were studied numerically by Wenqi Yuan et al. [12]. Using the ultrasonic transit time approach and HSC, T. Richter et al. [13] experimentally studied the diameter, trajectory, velocity, and tilt angle of argon bubbles that were rising in a zigzag pattern. Their research aimed to demonstrate that the ultrasonic transit time approach can ascertain the bubble dispersion and the increasing gas bubbles' diameter. Z. L. Yang et al. [14] used the Lattice Boltzmann technique to model bubble movement numerically examined the impacts of various physical factors and looked at the change in flow regime. According to their research, surface tension has no bearing on the changeover of flow regimes, whereas body force significantly affects the movement of bubbles. Similar-sized bubbles do not merge, and the flow regime will continue in such a situation. Brahim Mahfoud, Hibet El Mahfoud [15] studied the simulation of two-phase flow in an air-water column. They used (VOF) technique to explore the properties of bubble rise. A User Defined Function (UDF) imposed a custom boundary condition. They carried out all simulations both in two and three dimensions. They concluded that 3D simulations provide greater clarity in the case of the constant velocity influence on the flow, demonstrating that the velocity of bubble rise rises with an increase in air intake velocity. L. Pavlov, M. V. D'Angelo, and Mario Cachile [16] determined how the cell width W affected the bubble rise velocity, shape, and oscillations. They demonstrated that the mean relative velocity between the fluid and the bubble was independent of W. It addressed how the findings of an experiment with counterflow without lateral constriction compare to the analogy. They also demonstrated the presence of a regime for which the bubble shows solely form oscillations for sufficiently strong lateral confinements.

A bubble's zigzag rise usually happens in bubbly flows. The most significant causes of the turbulent flow produced in bubbly flows are thought to be the wake of the bubble and vortices shedding oscillations behind a bubble. As a consequence, an alternating lift force is generated [17]. Valentina Bello, Elisabetta Bodo, and Sabina Merlo [18] proposed a multi-parameter sensing system for simultaneous recognition of the fluid, based on its refractive index and the air bubble transit.

In this study, the dynamics of the bubbles are studied in 3D in three distinct sizes of water channels to show the more insightful picture of the rising air bubbles' characteristics. Two high-speed cameras are utilized in an experimental setup to examine the air bubbles ascent in the x-y (front view) and y-z (top view) planes. HSC is a powerful tool for examining the motion of bubbles. The VOF technique was used to simulate the two-phase flow numerically. The surface tension force was modeled as a constant value between two phases. RANS (Reynold Averaged Navier Stokes Equations) computations are performed using ANSYS FLUENT. MATLAB is used to draw the trajectories and as well as to provide the superimposed view of both the planes that are captured during experimental investigations. After that, those trajectories are presented in graphical form by using MS EXCEL and PLOT DIGITIZER (online tool).

Experimental Detail

Experiments were carried out in three distinct sizes of open rectangular acrylic glass channels. Details about the sizes of the three channels and water quantity filled in them are shown in the table 2.1. Two high-speed cameras were utilized to concurrently film the bubbles' rise in the channel in the x-y and x-z planes to corroborate the rising air bubble behavior. Both these cameras were set up to 200fps. A hole of 2mm is drilled in the center of each water tank along the x-axis. An injector with a diameter of ⌀2 mm and a length of 30 mm is inserted into the water tanks through this aperture. Through the use of an air injector, air is injected into the tank using an air storage device (compressor). A light source is used so that the digital high-speed cameras can record a clear video of the bubbles rising. Each of the four scenarios is individually investigated within each water channel. Schematics of the experimental setup are shown in Figure 2.2. Mass flow rates calculated through a digital flow meter are shown in Table 2.2.

Table 2.1: Channels sizes and water quantity

Bubble Profile	Mass Flow Rate (V_f) Channel-1: 200x80mm (Kg/s)	Mass Flow Rate (V_f) Channel-2: 400x160mm (Kg/s)	Mass Flow Rate (V_f) Channel-3: 400x240mm (Kg/s)
Single Bubble	0.0000350	0.0000555	0.000355
Two Bubbles	0.0000620	0.000260	0.000395
Series Of Bubbles	0.000350	0.000363	0.00160

Table 2.2: Mass Flow Rates

Sr.No.	Width (X=Zaxis)	Height (Y-axis)	Quantity of water filled for experiment in a channel.
Channel-1	80mm	200mm	180mm
Channel-2	160mm	400mm	200mm
Channel-3	240mm	400mm	200mm

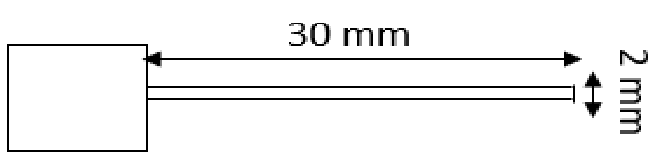


Figure 2.1: Submerged Needle schematic

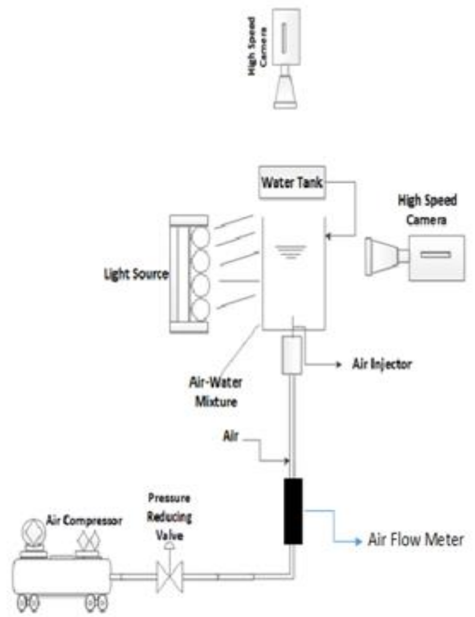


Figure 2.3: Water channels

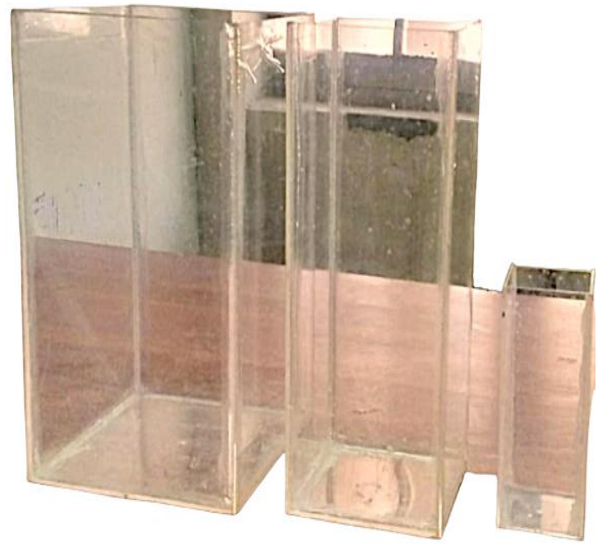


Figure 2.2: 3D Experimental setup schematic

Empirical Observations and Discussion

For the analysis of the recorded results, MATLAB coding is used for trajectories and superimposed view to analyze the behavior of the rising air bubbles. From the recorded films of the raised air bubbles and a jet in the water channel image processing techniques are required. Every image has a red, green, and blue color format of eight bits. It has to be changed from RGB to grayscale mode. The obtained picture has 256 gray levels. The levels are from 0 (white) to 255 (black). Eliminating bubble aspects from an original image straight away is a challenging task [19]. At first, the images were converted from RGB to grayscale. By applying threshold segmentation, the pictures which are converted to grayscale are changed over to binary mode. Thus, to analyze the trajectory of the bubbles. Four different cases of rising air bubbles—a single bubble, two bubbles, and a series of air bubbles, were investigated separately in the water channel throughout the experiments. Utilizing two HSC's at a speed of 200fps each case film was taped for 10 -15 seconds. Both HSC's were synchronized. The recorded films was changed over to different frames to get a more insightful view for study. Each bubble ascent scenario is replicated 8 to 10 times during experimentation to ensure comprehensive calibration and validation, thereby minimizing any potential ambiguity. The same findings were obtained when the experiments were repeated several times. All three cases in each channel were handled separately to observe the behavior of rising air bubbles. The prevailing experimental methodology for each channel is identical.

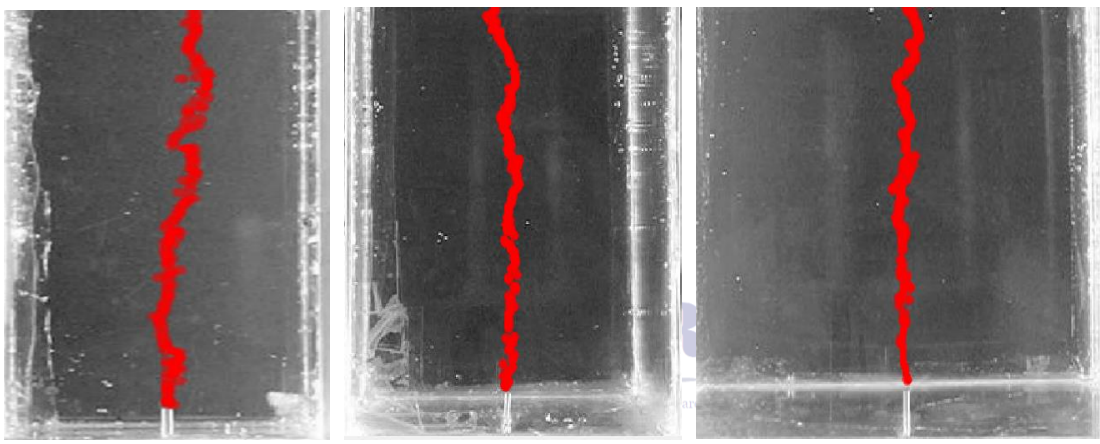
Case 1

Single Air Bubble Rise (X-Y) Plane

In the first instance, the flow rate was kept extremely low, allowing only one air bubble to be injected into the water channels separately to examine the rising characteristics of one air bubble in different sizes of water channels. The superimposed image of the single bubble rising in each water channel is seen in Figure 3.1. The trajectory of the single air bubble rise in the water channels is shown by the red line in Figure 3.2, which follows a zig-zag path instead of a straight climb. Ascent of a single bubble in the x-y plane is depicted graphically in Figure 3.3(a-c).

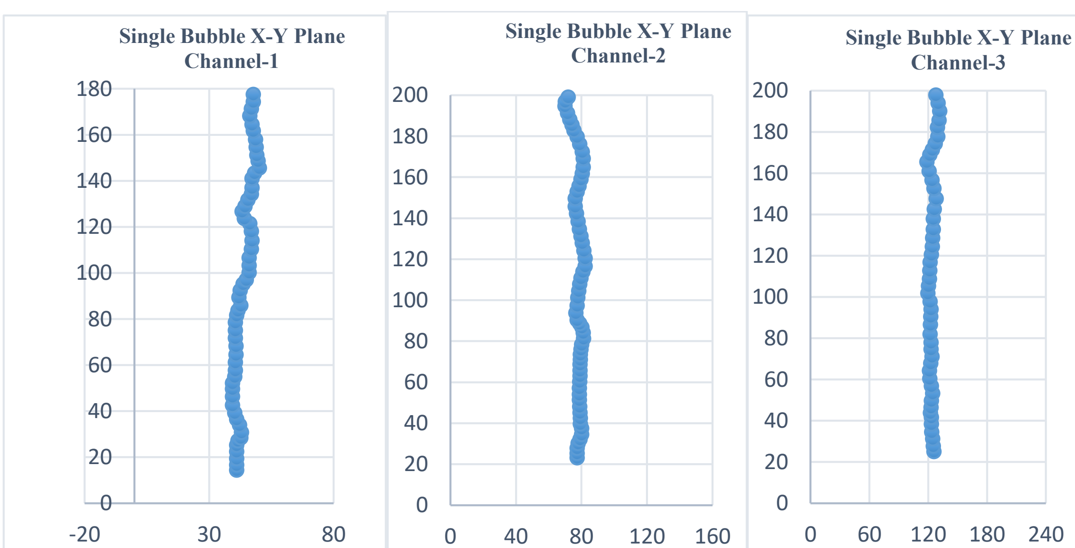
Channel-1 Channel-2 Channel-3

Figure 3.1: Superimposed View of single bubble rise in water channels.



Channel-1 Channel-2 Channel-3

Figure 3.2: Trajectories of single bubble rise in water channels.



(a) (b) (c)

Figure 3.3(a-c): Graphical depiction of single bubble rise in X-Y Plane.

Single Air Bubble Rise
(X-Z) Plane
As the HSC camera recorded the bubble's movement in the (x-y plane), at the same instant another HSC camera positioned at the top of the water channel simultaneously recorded the bubble's movement in the (x-z plane). Nothing changes in terms of the flow conditions, such as flow, etc. figure 3.4 depicts the superimposed view of the rising air bubble. The superimposed images shows the rise of the single bubble in water channels which is again a zig-zag ascent in x-z plane rather than following a straight route. Figure 3.5 depicts the graphical representation of bubble rise in x-z plane.

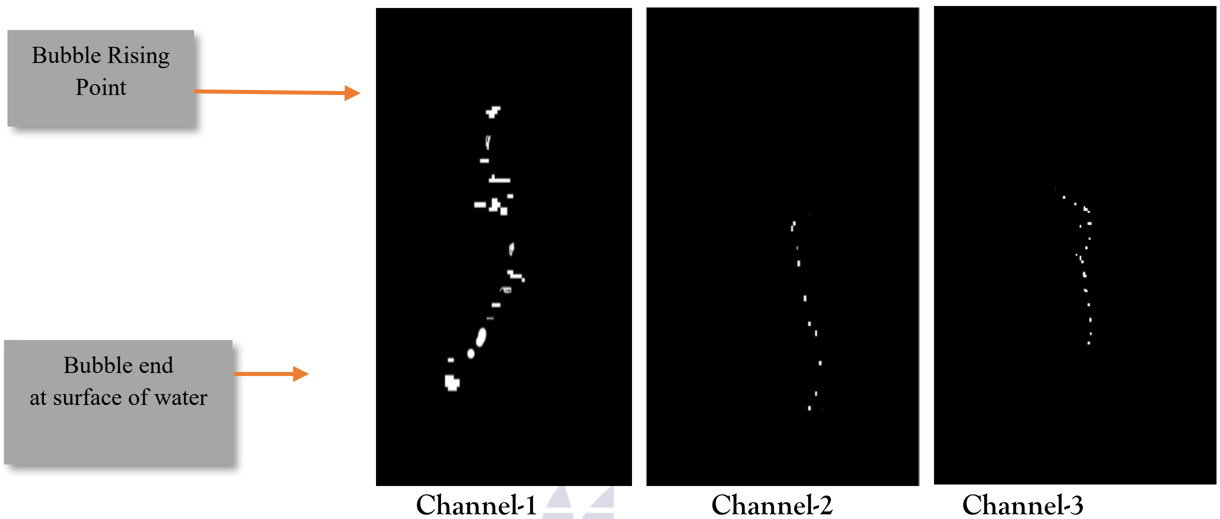
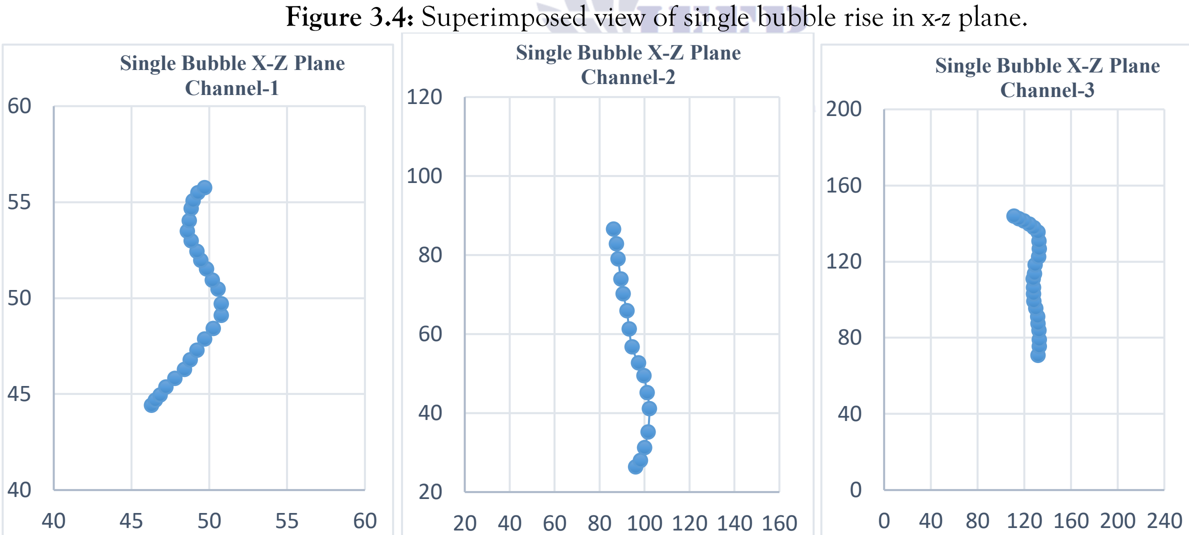


Figure 3.4: Superimposed view of single bubble rise in x-z plane.



(a) (b) (c)

Figure 3.5: Graphical depiction of single bubble rise in x-z plane.

Case 2
Two Air Bubbles Rise (X-Y) Plane
The flow rate was then increased to introduce two air bubbles in the water channels. Two air bubbles rising at different frame rates in the water channels are shown by the superimposed view in Figure 3.6. Figure 3.7 displays the red line representing the two rising air bubbles' trajectory. Figure 3.8(a-c) shows the path's graphical representation. Instead of following a straight path, both bubbles ascend in each channel in a zigzag pattern.

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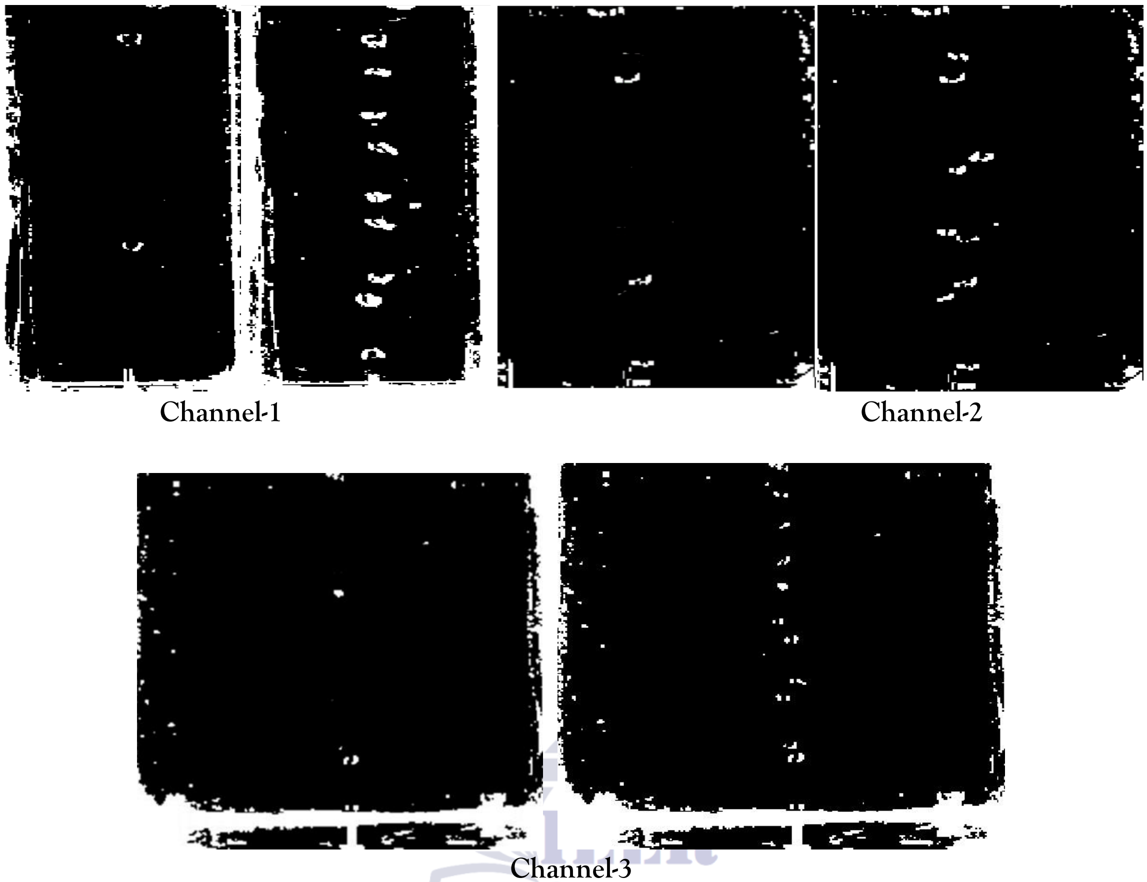


Figure 3.6: Superimposed View of single bubble rise in water channels

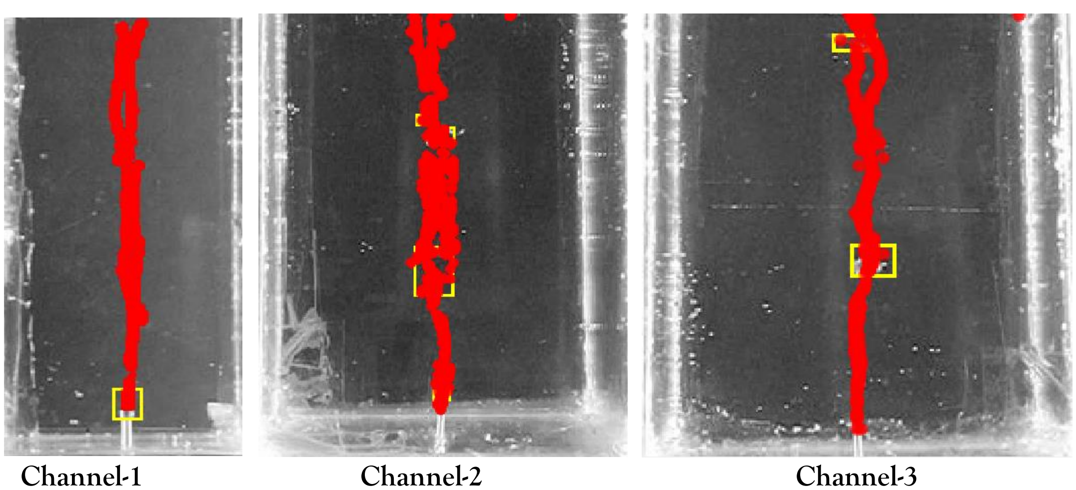
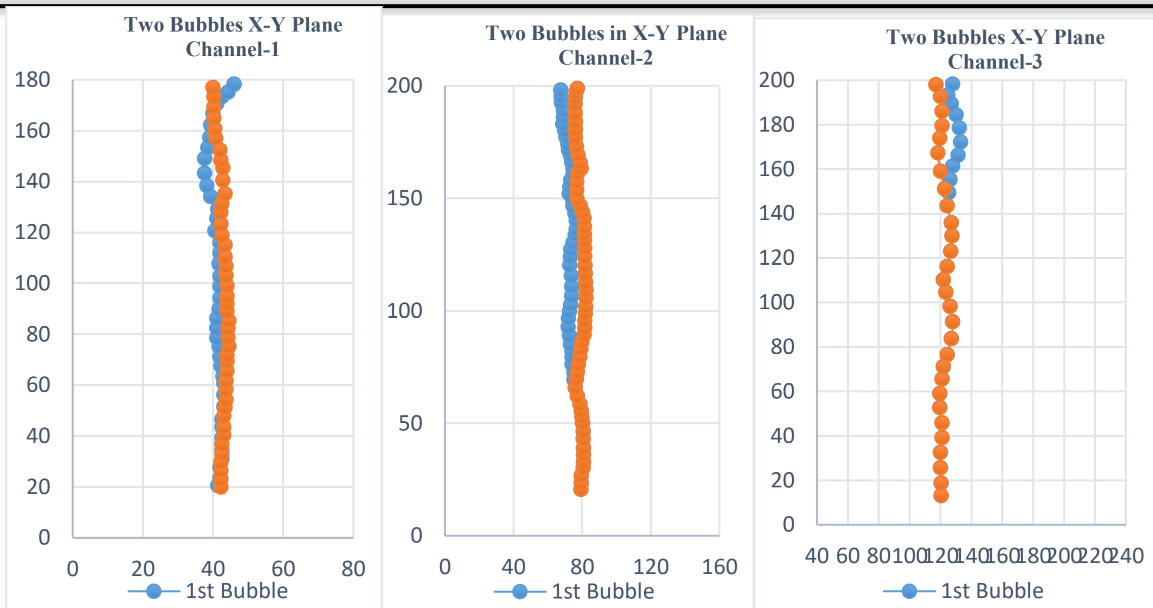


Figure 3.7: Superimposed View of single bubble rise in water channels.

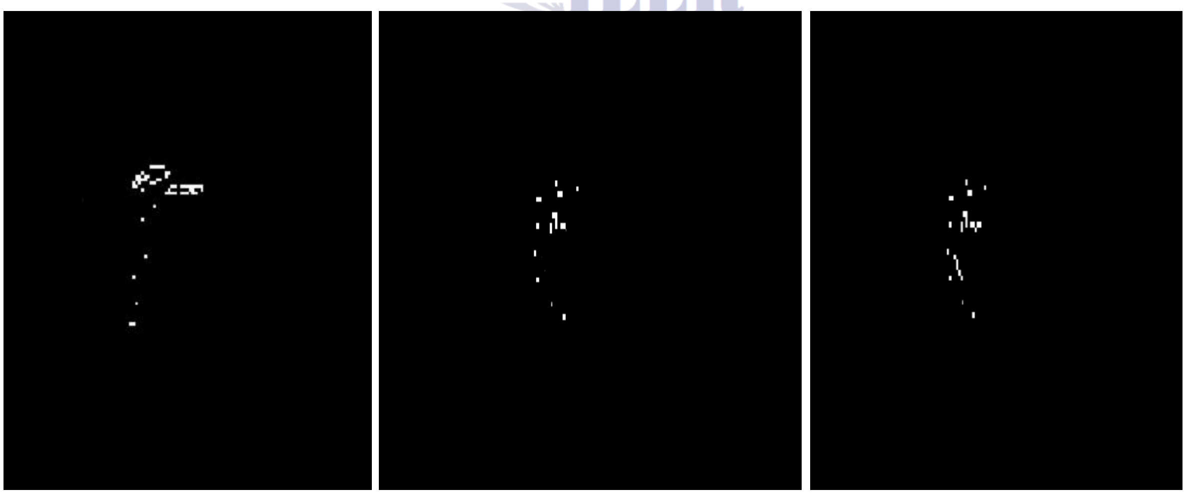


(a) (b) (c)

Figure 3.8(a-c): Graphical depiction of two bubble's rise in the X-Y Plane.

Two Air Bubble's Rise (X-Z) Plane

In the meantime, the H.S.C. camera captured the movement of two rising bubbles from the x-z plane, and the outcome was a zigzag rather than a straight rise, similar to the x-y plane. It is clear from the superimposed view in Figure 3.9 as well as from the graphical illustration shown in Figure 3.10 (a-c).



Channel-1 Channel-2 Channel-3

Figure 3.9: Superimposed view of two bubble's rise in x-z plane.

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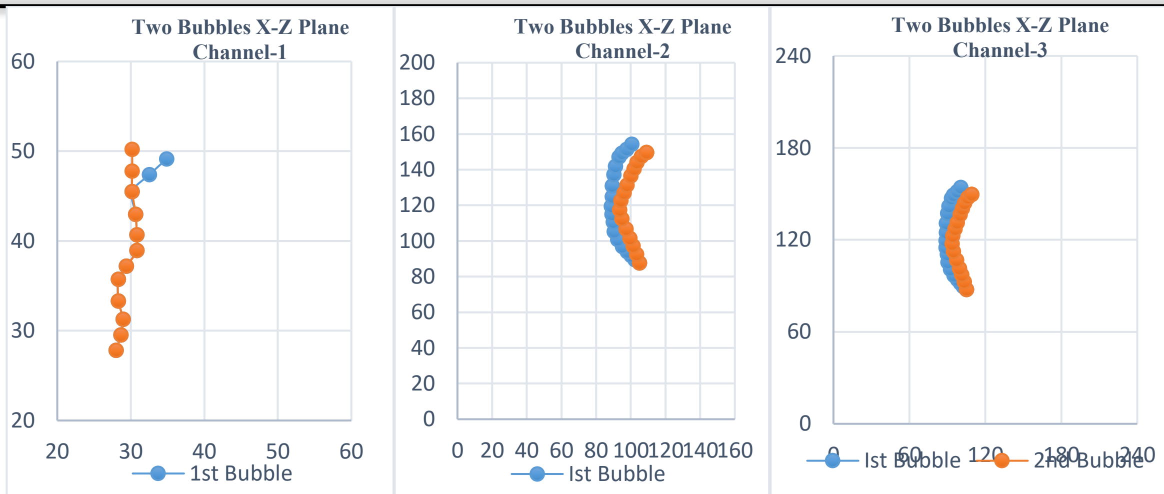

(a) (b) (c)

Figure 3.10: Graphical depiction of two bubble's rise in the x-z plane.

Case 3

Series of air Bubbles Rise

(X-Y) Plane

In alignment with the observations from the previous two cases, where a zig-zag pattern of air bubbles was noted when the flow rate was further increased as specified in Table 2.2 to generate a series of bubbles in the channels. The resulting data indicated that a series of bubbles demonstrated similar behavior. The series of bubbles formed within the water channels exhibited a zig-zag path rather than a rectilinear path. Figure 3.11 shows the superimposed view of the series of bubbles rising while in Figure 3.12 red line shows the trajectory of the series of bubbles rising in each channel

Channel-2 Channel-2 Channel-3

Figure 3.11: Superimposed view of series of bubble rise in x-y plane.

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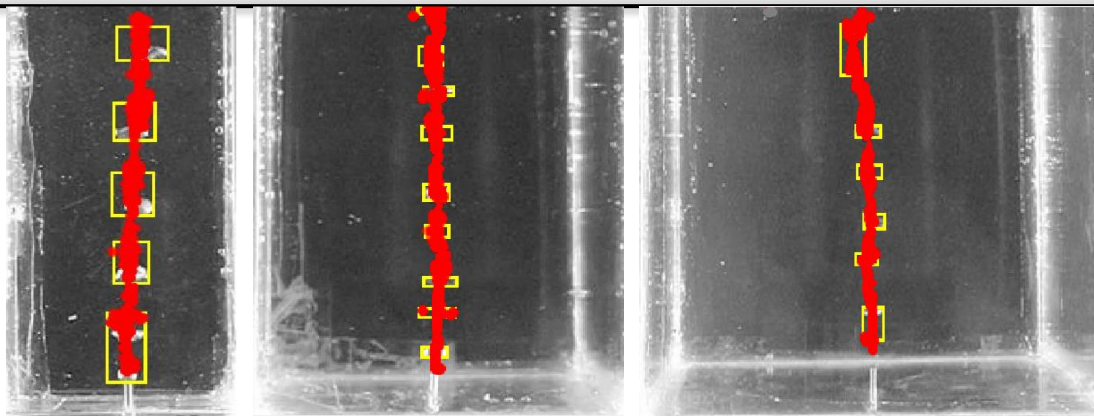
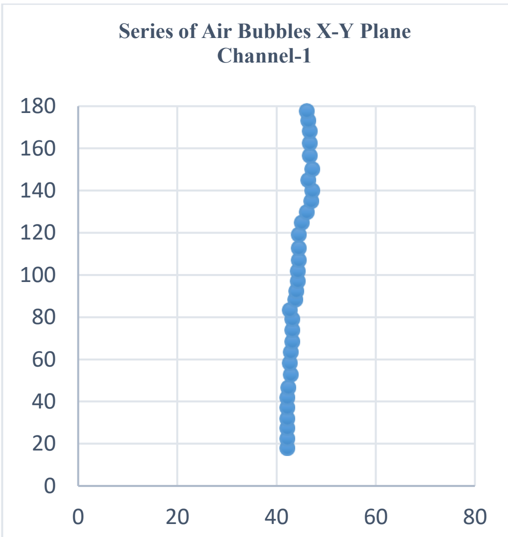
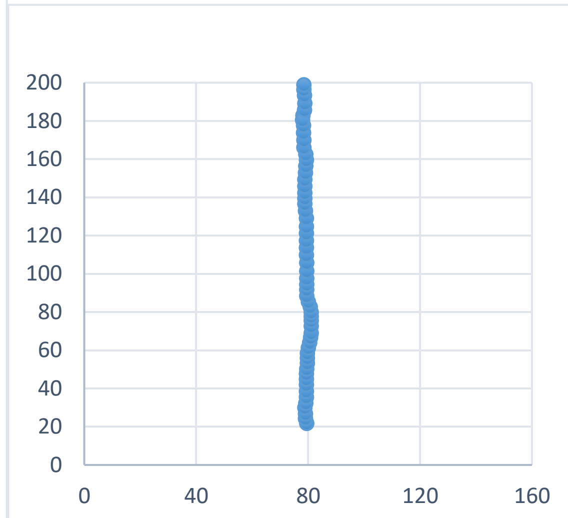


Channel-1 Channel-2 Channel-3

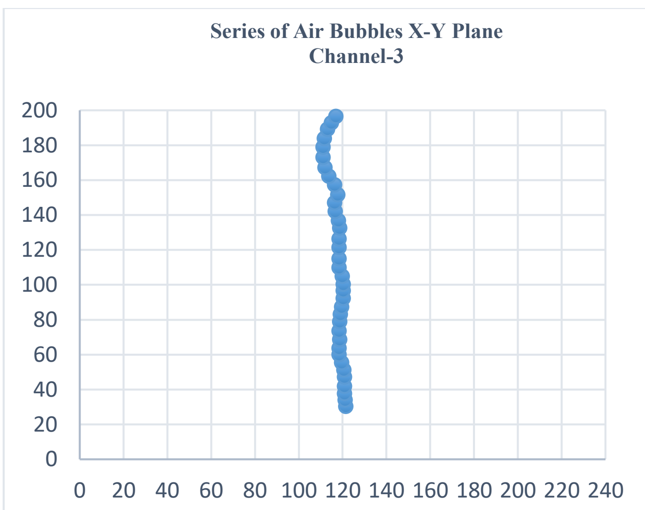
Figure 3.12: Trajectories of series of bubble rise in x-y plane in channels.



(a)



(b)



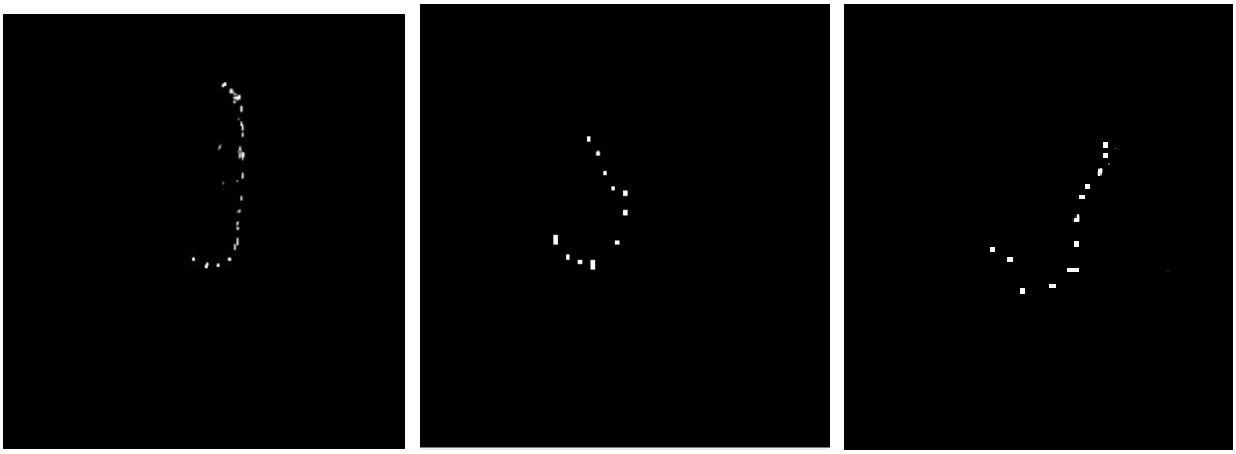
(c)

Figure 3.13: Graphical depiction of a series of bubble's rise in the x-y plane.

Series of air Bubbles Rise

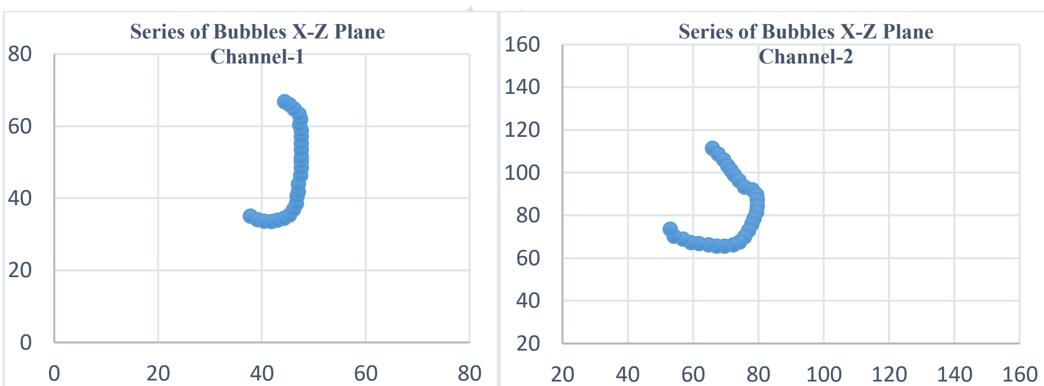
(X-Z) Plane

The series of raised air bubbles captured in the (x-z) plane also displayed a zig-zag pattern. The graphical ddd view and superimposed images verify that the bubble row was not ascending in a straight line in the channels

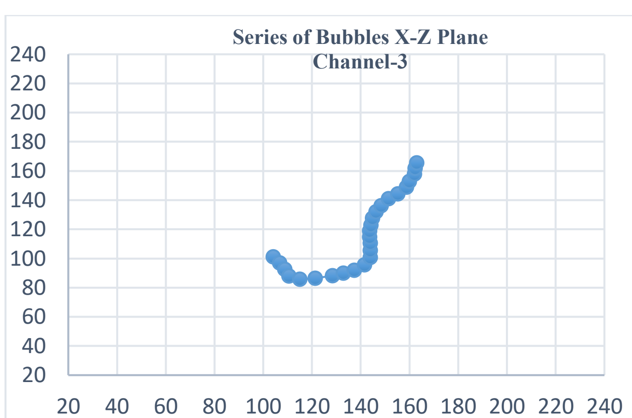


Channel-1 **Channel-2** **Channel-3**

Figure 3.14: Superimposed view of series of bubble's rise in x-z plane.



(a) (b)



(c)

Figure 3.15: Graphical depiction of series of bubble's rise in x-z plane.

Numerical Simulations

Utilizing RANS computations, numerical simulations were performed to investigate the rise trajectory of air bubbles in water. Two-dimensional (2D) vertical rectangular water channels that matched the dimensions of the experimental water channels were used in the investigation. Table 2.1 provides the physical details of the water channels. The lower portion of the water channels is filled with water, the upper segment is initialized with air. A 2 mm diameter by 30 mm long needle was modelled and placed vertically in the middle of the channel.

The physical properties of air and water used are given in Table 2.3.

Table 2.3: Properties of the water and air used in the simulation

Phase	Viscosity (Pa.s)	Density (Kg/m^3)	Surface tension (N/m)
Water	0.001	998.2	0.073
Air	1.789x10^{-5}	1.225	-

Boundary Conditions

The bottom and side edges of the channels are modeled as non-slip walls, while the top edge is set as an outflow boundary condition. An interface is located at 180 mm in channel 1 and 200 mm in channels 2 and 3 along the y-axis. In the operating conditions for FLUENT, the operating pressure is set to 101,325 Pa, and the gravitational force (g) is directed downward at -9.81 m/s^2 along the y-axis.

Numerical Approach

To simulate the bubble rise trajectories in water, Ansys Fluent 20 is utilized. Simulations are run transiently. Air is considered to be the secondary (dispersed) phase and water is the primary (continuous) phase. Turbulence closure is achieved by using the K-Ω model. Moreover, implicit body force treatment is applied, which strengthens the solution. The pressure-implicit with splitting of operators (PISO) pressure-velocity coupling scheme which is the component of the SIMPLE family of algorithms, is used for pressure velocity coupling. It is appropriate for short-term calculations and offers a fast convergence rate without sacrificing precision. Pressure is discretized via the PRESTO system. Making other propositions (such as second-order schemes or linear ones) is not advisable as they might lead to significant divergence or slower convergence. Momentum and pressure were set to 0.4 and 0.6, respectively, under relaxation factors. All simulations were run with a time step of 0.0001s. The “courant number” was set to be 0.255 and ambient pressure was adjusted to 101.325kPa. The mass flow rates used in the simulations were the same as those recorded during the experimental investigation, as presented in Table 2.2.

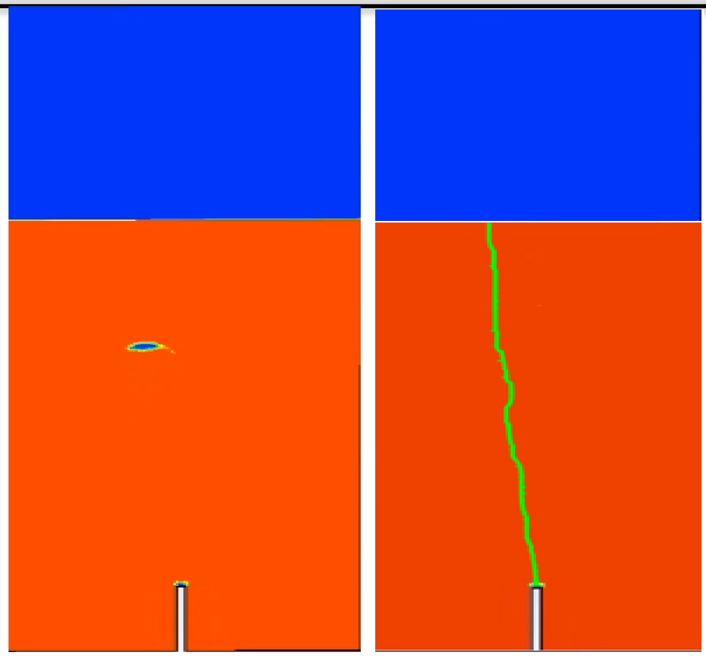
Quantitative Results and Discussion

The results of the two-dimensional numerical simulations for each water channel indicate that air bubbles follow a zig-zag trajectory instead of ascending in a straight line, consistent with the experimental findings. The green line clearly illustrates the bubbles' ascent within the water channels, demonstrating the zig-zag path, which aligns with the experimental observations. The mass flow rate used in the numerical simulation was the same as that recorded during the experimental investigation. All results presented here were obtained using the SST model.

Case-1

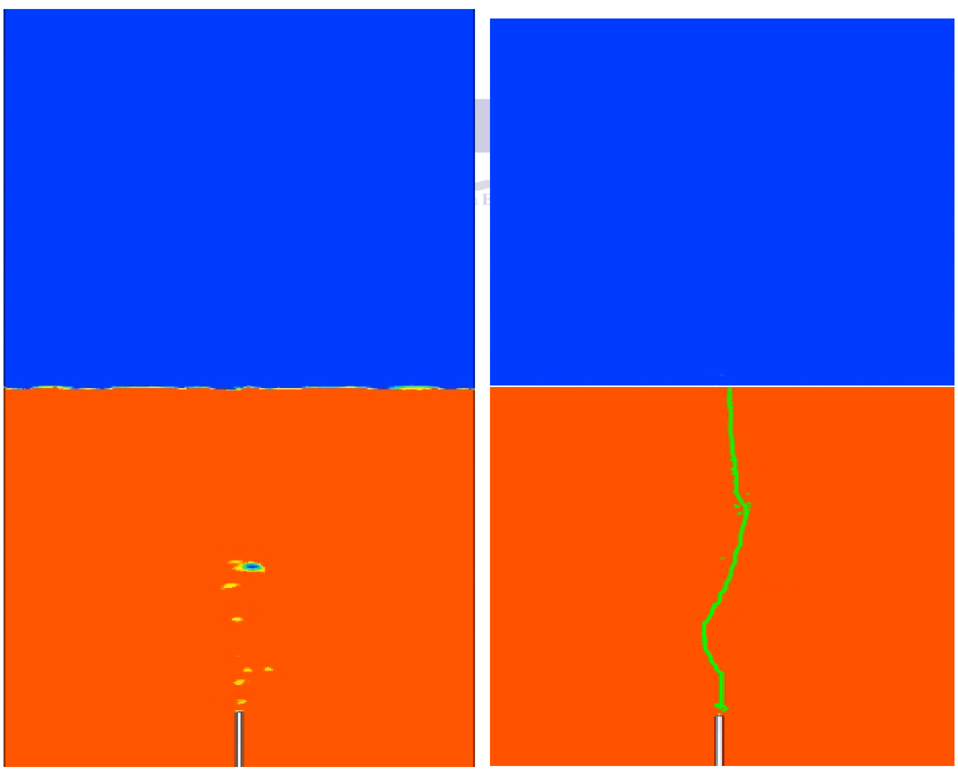
Single Bubble Rise

The observed numerical results indicate that a single air bubble in each water channel exhibits a zig-zag ascent rather than a straight vertical rise. The green line illustrates the trajectory taken by the single bubble during the rise.



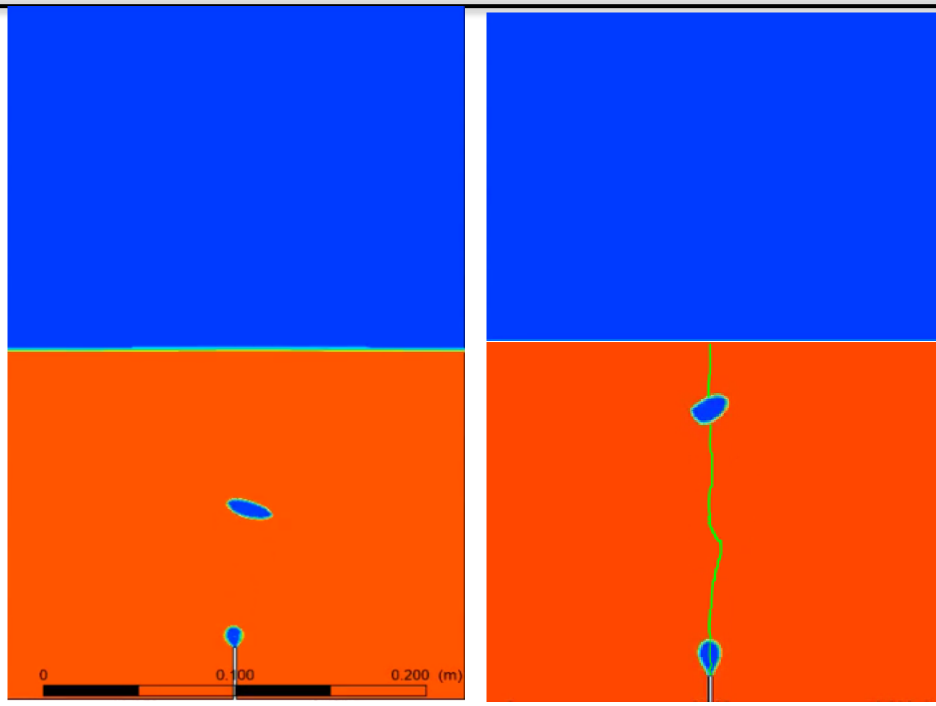
A B

Figure 7.1: (a) Rise of single bubble in channel-1 (b) Trajectory of the single bubble rise in channel-1.



A B

Figure 7.2: (a) Rise of single bubble in channel-2 (b) Trajectory of the single bubble rise in channel-2.



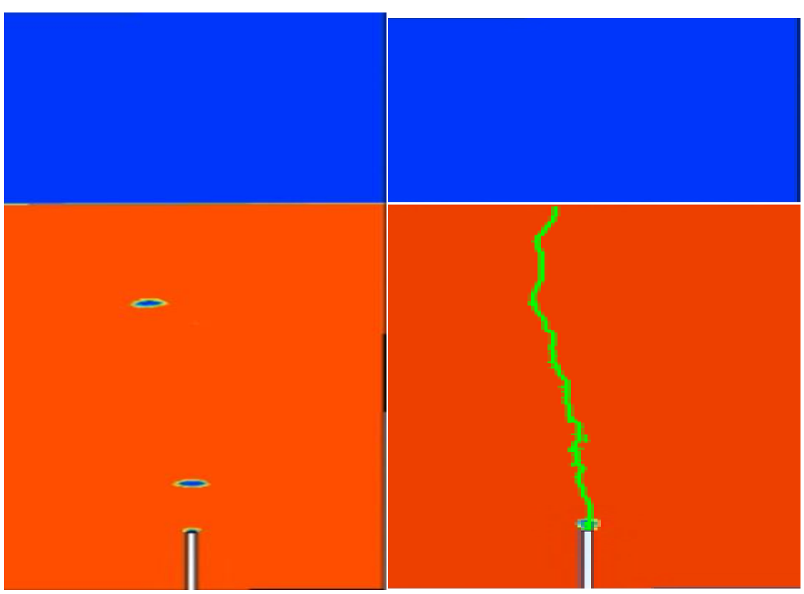
(a) (b)

Figure 7.3: (a) Rise of single bubble in channel-3 (b) Trajectory of single bubble rise in channel-3

Case-2

Two Bubble's Rise

When two bubbles are simulated rising in each channel, they exhibit the same behavior as a single bubble, demonstrating a similar zig-zag ascent pattern rather than straight rise. From fig.7.5 & 7.6 we see that as the vertical water channel size increases, the rising paths of two air bubbles diverge due to disturbances like turbulence and pressure gradients caused by the first bubble. These flow variations result in the second bubble following a different or irregular path.



A B

Figure:7.4 (a) Rise of two bubbles in channel-1 (b) Trajectory of two air bubbles rise in channel-1

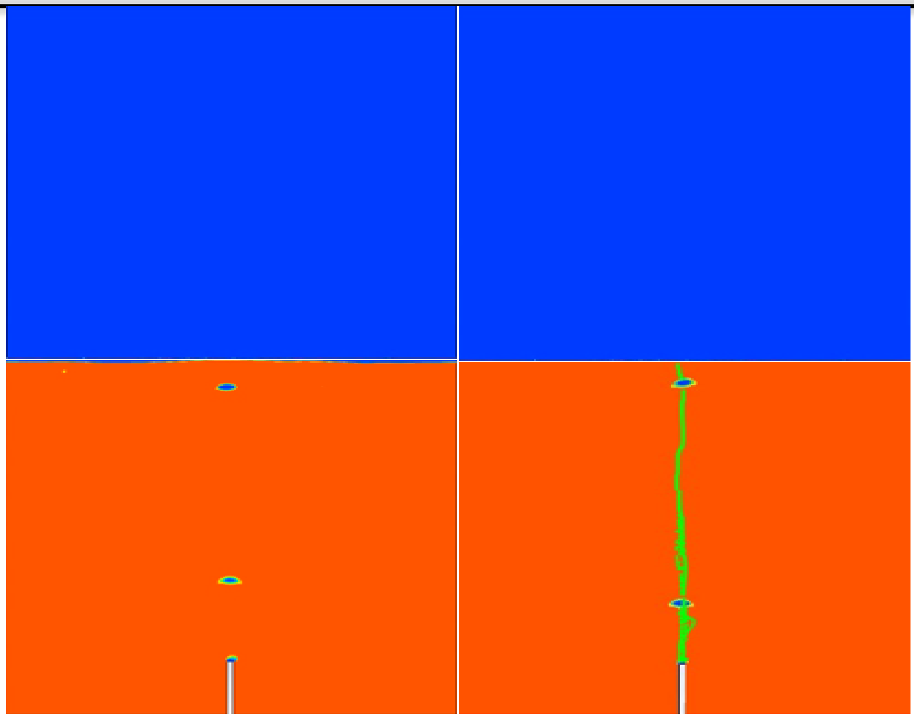

(a) (b)

Figure 7.5 (a) Rise of two bubbles in channel-2 (b) Trajectory of two air bubbles rise in channel-2

Conclusion

Observations from both the x-y and x-z planes indicate that the bubble follows a zig-zag trajectory rather than rising in a straight, rectilinear path. The shape of the bubble transitions from round to elliptical as the channel's size changes, which is attributed to increased inertial forces, such as drag, acting on the bubbles. The mass flow rate experiences slight variations due to the changing channel width. A larger channel size results in reduced velocity gradients at the boundaries, lowering frictional drag and allowing the fluid to flow more easily through the channel. Numerical simulations align with experimental results, showing that the bubbles follow a similar path, regardless of the influence or proximity of the channel walls. As the bubble rises, its motion becomes more irregular due to changes in channel size, which affect local flow conditions like pressure gradients and turbulence intensity. These variations lead to complex interactions between the fluid and the bubble, causing fluctuations in the forces acting on the bubble. The SST model successfully predicted the same behavior as observed in the experiments.

Declarations

a) **Conflicts of Interest/Competing Interests**

The authors declare no conflicts of interest.

b) **Authors' Contributions**

All authors contributed equally to this manuscript's conceptualization, methodology, analysis, and writing.

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